Abstract: Height control of the aircraft is one of the most important tasks to be solved during preliminary design of the flight phases. From the point of view of the flight safety it is also important to design a safe automatic flight control system. There is a big challenge in this field of derivation of the flying and handling qualities of the remotely piloted airplanes, such as unmanned aerial vehicles, including both airplanes and helicopters. The paper highlights this problem giving some suggestion for the solution. These results are very early and pilot – they belong to a research program called “Computer Aided Analysis and Design of the Vehicle Systems” lead at Miklós Zrínyi National Defense University. Design methods applied during series controller synthesis are both classical, and modern optimal ones. Design requirements are defined in military specifications of MIL–F–8785C, and MIL–STD–1797A. Solution of the closed loop control system design, and analysis problem is supported by MATLAB® computer package supplemented with its necessary toolboxes.

Keywords: height control of the aircraft, control system design, LQR method.

1. INTRODUCTION

Height control of the UAV is important from many aspects. The first reason is that it is important phase of the automated control allowing monitor the ground, or the water surfaces. Height control consists of many possible flight regimes, such as height stabilization, or model following. In this particular case height changes as pre-defined deterministic flight parameter and, mission is to follow this reference value.

These regimes are applied during landing, during flare phase of the landing, in emergency landing, and, during avoiding obstacles on the ground. There is no question about this, and understanding importance of this topic authors are investigating the preliminary design of the height control system for the hypothetical UAV aircraft, having slow dynamics. During control system preliminary synthesis requirements are taken from military standards given for piloted airplanes. The reason for this is lack of flying and handling qualities for the UAV aircraft. After design we will show results of the computer simulation both in frequency, and time domain.

2. BRIEF HISTORY & LITERATURE OVERVIEW

Basic methodology of modern control systems based on state space representation is outlined in references of [1,2,3,5,7]. This fundamental literature was used during preparing this article. Automatic control systems and its applications are discussed in [4,5,13,16]. Dynamics of flying objects and its mathematical description is given in [6,15]. Last decades application of CAD technology in classical, in modern, and, in post-modern control systems analysis and design became evidence. There are many excellent textbooks propagating computer aided design and analysis such as [10,11,12,13,16,17,18]. There are many early [8,9] and late [14,19] military standards about automatic flight control systems, and lots of flying and handling qualities are summarized here. The only bothering thing arising here is that these standards are for airplanes piloted by the
human operator on the board. However, if to think over problems of design control systems for the manned, and for the unmanned aircraft, it is easy to see that requirements of manned aircraft are stricter, and they can be applied also for UAV systems.

Computer aided analysis will be carried out to test whether automatic flight control system is able to minimize unwanted effects from environmental disturbances, namely, atmospheric turbulences and its effects on flight of the hypothetical UAV will be analyzed. Mathematical models and theirs computer aided simulation is outlined in [20]. Computer code used for this task is a new MATLAB embedded file created by the authors.

3. CONTROL SYSTEM DESIGN USING LQR METHOD

Dynamics of the LTI system can be defined using the following state and output equations [6,7,10,11,12,13,16]:

\[
\dot{x} = Ax + Bu, \quad y = Cx + Du
\]  
(01)

where A represents a state matrix, B is input matrix, C is output matrix, and finally, D is a direct feedforward matrix, respectively, and x is a state vector, and u represents input vector. Block diagram of the closed loop control system built by equations (01) can be seen in Figure 1, when feedforward matrix is zero, i.e. D = 0.

\[
\begin{align*}
&x_{ref} \\
&\sum\quad B \\
&\sum\quad \dot{x} \\
&\sum\quad J \\
&\sum\quad C \\
&A \\
&K
\end{align*}
\]

Fig. 1 Block Diagram of the Control System

Optimal control law can be determined evaluating the following integral performance criteria [10,11,12]:

\[
J = \frac{1}{2} \int_{0}^{\infty} \left( x^T Q x + u^T R u \right) dt \rightarrow \text{Min} \quad (02)
\]

In this cost function main design parameters are weights on state vector Q \( \geq 0 \), i.e. it is a positive semi-definite matrix, and weights on control vector R > 0, i.e. it is a positive definite weighting matrix. If Q is very large relative to R one can get a closed loop system response with large overshoots. If R is chosen to be very large relative to Q control system has smaller actuators, electric motors, amplifier gains and other devices. During controller synthesis weighting matrices can be derived using the so-called inverse square rule.

The LQ optimal control problem can be solved using wide variety of techniques. Let us consider method of Euler-Lagrange equations, Hamilton-Jacobi-Bellman theory and Pontriyagin's minimum principle. Firstly let us define the so-called Hamiltonian matrix:

\[
H(x,\lambda, t) = \frac{1}{2} \left( x^T Q x + u^T R u \right) + \lambda^T (Ax + Bu)
\]  
(03)

where \( \lambda \) is the Lagrange multiplier, H is the Hamiltonian matrix.

It is well-known that Pontriyagin’s minimum principle states that optimal state and control trajectories must satisfy the following equations [7]:

\[
\frac{\partial H}{\partial \lambda} = \dot{x}; \quad \frac{\partial H}{\partial x} = -\lambda; \quad \frac{\partial H}{\partial u} = 0
\]  
(04)

Using rules for differentiation of matrices and vectors equations of (04) can be rewritten in the following manner

\[
\dot{x} = Ax + Bu, \quad x(0) = x_0
\]  
(05)

\[
\dot{\lambda} = Qx + A^T\lambda, \quad \lambda(T) = 0
\]  
(06)

\[
u^o = -R^{-1}B^T\lambda
\]  
(07)

Formula (07) defines the optimal control law of the closed loop control system. The coupled equations (05), (06) and (07) are main equations for the so-called ‘two point boundary value problem’ (TPBWP).

Substituting equation of control law (07) into state equation (05) results in following formula:

\[
\begin{bmatrix}
\dot{x} \\
\dot{\lambda}
\end{bmatrix} =
\begin{bmatrix}
A & -BR^{-1}B^T \\
-Q & -A^T
\end{bmatrix}
\begin{bmatrix}
x \\
\lambda
\end{bmatrix} = H
\]  
(08)

Let us make the following substitution in equation (08):

\[
\lambda = Px,
\]  
(09)
where \( P \) is the so-called cost matrix. Differentiating both sides of equation (09) with respect to time and considering equations (05) and (07) the following equation can be derived:

\[
\frac{d\lambda}{dt} = \frac{dP}{dt} x + P \frac{dx}{dt} = x + P A x - P B R^{-1} B^T P x
\]

\( P B R^{-1} B^T P x = -Q x - A^T P x \)

The sufficient condition for optimal control is that \( P \) must satisfy the following Ricatti differential equation:

\[
- \frac{dP}{dt} = A^T P + P A + Q - P B R^{-1} B^T P, P(T) = 0
\]

(10)

Solution of the optimal controller synthesis problem using Ricatti equation in control theory is regarded as the finite time problem. This solution results in the linear time varying static controller of the feedback, i.e.:

\[
u^*(t) = -K(t)x(t), K(t) = R^{-1} B^T P(t)
\]

Equation (11) is a nonlinear first order matrix differential equation, which has to be solved backwards in time. During solution of the infinite time LQR problem it is considered that \( T \rightarrow \infty \). It is obvious that under mild conditions cost matrix \( P \) can be considered as constant and solution of Ricatti equation results in the asymptotically stable closed loop control system. In this particular case equation (11) can be rewritten as:

\[
A^T P + P A + Q - P B R^{-1} B^T P = 0
\]

and, the optimal control vector can be derived as:

\[
u^0(t) = -Kx(t), K = R^{-1} B^T P
\]

Equation (13) is known as algebraic Ricatti equation (ARE). Conditions defined by equations (13) and (14) are necessary and sufficient for existence of the optimal controller, which will asymptotically stabilize the closed loop control system. Thus, procedure of optimal control law synthesis consists of the following two steps: 1. solution of the ARE – equation (13) – in order to find constant cost matrix \( P \); 2. substitution cost matrix \( P \) into equation (14). The resulting feedback gain matrix \( K \) is an optimal for the chosen weighting matrices \( Q \) and \( R \).

4. A NUMERICAL EXAMPLE

This chapter deals with methodology of the closed loop automatic flight control system preliminary design, and analysis, thus, gives main guideline for control engineers. The closed loop automatic flight control system will be synthesized using optimal Linear Quadratic Regulator (LQR) design method outlined in deep details in Chapter 3.

Let us consider automatic flight control system used for height control system, which is given in Figure 2. The block diagram is general, and suggested to use it for height control of the UAV first time by the authors.

![Fig. 2 Block Diagram of the Classical Height Control System](image)

Block diagram given in Figure 2 represents the full state feedback control system. Dynamics of the UAV is represented by the following transfer function:

\[
Y(s) = \frac{v_{ac}(s)}{\delta_E(s)} = \frac{A}{1 + sT} = \frac{5}{1 + 2s}
\]

(15)

where \( v_{ac}(s) \) is airspeed of the aircraft, \( \delta_E \) is elevator angular deflection, \( H_r(s) \) is reference signal of the altitude, \( H(s) \) is actual value of the altitude, and finally, \( \Delta H(s) \) is error signal of the automatic flight control system. Parameters of the transfer function (15) are chosen fully theoretically, however transfer function must represent a slow dynamics with pure dynamic characteristics. Airspeed sensor is represented by transfer function of \( Y_{s_1}(s) \), and its transfer function is one of the design parameters. Barometric altimeter is given with its transfer function of \( Y_{s_2}(s) \), which will be supposed to have unity gain. Note that this model is simplified to that of the possible simplest one. The series compensator is represented with transfer function of \( Y_c(s) \)
and, it is the second design parameter to be found.

4.1. Analysis of the Uncontrolled UAV Dynamics. The open loop UAV was tested both in time and in frequency domain. Result of the computer simulation can be seen in Figures 3.1…3.4.

Figure 3.1 shows impulse response, Figure 3.2 shows step response of the hypothetical UAV dynamics. From these figures it is easily can be seen that UAV dynamics has first order term feature. Response functions are exponential ones, the UAV responses to inputs very slowly. Regarding dynamic performances, flying and handling qualities given in [8,9,14,19] it can be said that dynamic performances shown by UAV dynamics out of the required ranges. Figure 3.3 represents transient response of the open loop system to square signal having period of time of $T = 10 \text{ sec}$, or frequency of $f = 0.1 \text{ Hz}$. As this figure shows, having slow dynamics UAV is unable to follow the reference input of the square signal. Thus, UAV needs flight control system making it able to achieve fast responses, in other words, fast maneuvers. Analysis in the frequency domain shows that UAV has good dynamic performances, i.e. gain margin is $G_m = \infty$, and phase margin is $P_m = 102^0$. From this argue comes out the evidence of the need of the automatic flight control system to control the height of the UAV. There are many classical and modern methods, which can be used for control system preliminary design. Regarding problems of UAV systems’ design, for example design of the energy system, the more feasible methods are limited to optimal design methods.

4.2. Gain Selection Using LQR Design Method. Before start design of the height control system shown in Fig. 2. Let us define dynamic performances of the closed, and open loop dynamic characteristics and requirements to be as follows in Table 1 [14,19]:

<table>
<thead>
<tr>
<th>Time Domain Requirements</th>
<th>Frequency Domain Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady-state settling time: $t_{ss} \leq 2 \text{ sec}$</td>
<td>Gain margin: $G_m \geq 10 \text{ dB}$</td>
</tr>
<tr>
<td>Damping ratio: $0.7 \leq \xi \leq 0.8$</td>
<td>Phase margin: $P_m \geq 45^0$</td>
</tr>
</tbody>
</table>

Before using LQR design procedure to find system parameters, closed loop control system
given in Figure 2 in complex frequency domain, it must transferred to the following one, given in the time domain. Block diagram of the time domain closed loop control system given in Figure 4.

![Figure 4 State Space Model of the Closed Loop Control System](image)

Using basic equations of the inverse Laplace-transformation, matrices, and vectors used in Figure 4 easily can be found to be as follows:

\[
\dot{x} = Ax + Bu = \begin{bmatrix} \frac{1}{T} & 0 \\ 1 & 0 \end{bmatrix} v_{ac} + \begin{bmatrix} 0 \\ 2.5 \end{bmatrix} \delta_E \]

\[
y = Cx + Du = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} v_{ac} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \delta_E
\]

Static feedback gain matrix \( K \) is as follows:

\[
K = \begin{bmatrix} Y_{s_1} \\ Y_c \end{bmatrix}
\]

(17)

During controller synthesis for closed loop control system given in Figure 4 first step must be done is selection of the weights of the integral criteria defined by equation of (02). There are many well-known methods for selection of the weights, such as inverse square rule, unity weighting, and, finally, heuristic set of weights. Having no information about limitations of the automatic flight control system we will choose for the first step unity weighting for matrices \( Q \), and \( R \), respectively:

\[
Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; R_1 = \eta = 1
\]

(18)

For the unity weighting matrices defined by equation of (18) the static feedback gain matrix is as follows [10,11,12,17,18]:

\[
K_1 = \begin{bmatrix} 1,1565 & 1 \\ 1,1565 & 1 \end{bmatrix}; Y_{s_1} = 1,1565; Y_c = 1
\]

(19)

The closed loop control system for given static controller was tested for dynamic performances.

### Table 2 Dynamic performances of the closed loop control system

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Damping ratio, ( \xi )</th>
<th>Natural frequency, ([\text{rad/s}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1,08</td>
<td>1</td>
<td>1,08</td>
</tr>
<tr>
<td>-2,31</td>
<td>1</td>
<td>2,31</td>
</tr>
</tbody>
</table>

Step response of the control system can be seen in Figure 5. From this figure it is evident that steady-state settling time is \( t_{ss} \approx 3,3 \text{sec} \), and, damping ratio has unity value. The response of the closed loop control system to the reference unity gain signal is exponential one.

![Fig. 5 Transient response of the height control system](image)

At the next stage of the control system design find heuristically the appropriate set of weighting matrices \( Q \), and \( R \) providing for the closed loop control system, in this particular case it is a height control system of the UAV, dynamic performances defined before. After several attempts of the control system synthesis and analysis carried out by the authors following set of weighting matrices shall be used for the controller synthesis:

\[
Q_2 = \begin{bmatrix} 0,1 & 0 \\ 0 & 15 \end{bmatrix}; R_2 = r_2 = 15
\]

(20)

For this set of matrices static state feedback gain is as follows [17,18]:

\[
K_2 = \begin{bmatrix} 0,7201 & 1 \\ 0,7201 & 1 \end{bmatrix}; Y_{s_1} = 0,7201; Y_c = 1
\]

(21)
For this particular case dynamic performances were found and put into Table 3.

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Damping ratio, $\zeta$</th>
<th>Natural frequency, $[\text{rad/s}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.15 + 1.08i$</td>
<td>0.727</td>
<td>1.58</td>
</tr>
<tr>
<td>$-1.15 - 1.08i$</td>
<td>0.727</td>
<td>1.58</td>
</tr>
</tbody>
</table>

The closed loop step response was analyzed. Result of the computer simulation can be seen in Figure 6.

Fig. 6 Transient response of the height control system.

From Figure 6 it is easily can be seen that $t_{ss} \approx 1.8\text{sec}$, and damping ratio is $\zeta = 0.727$. Thus, time domain dynamic performances of the closed loop control system are in ranges with those of defined in Table 1.

Comparison of the two systems designed using special sets of weighting matrices can be performed using Figures 7.1 and 7.2. Figure 7.1 shows transient responses of the inner loop, which is loop for airspeed of the aircraft.

Fig. 7.1 The inner loop transient response

Fig. 7.2 The outer loop transient response

It is evident from this figure that change of elements of weighting matrices to those of given in equation (20) result in system, which behaves more oscillatory than previous one. However, this allows to closed loop control system to react faster to reference unity input measured in the height of the flight.

Finally, let us find Bode diagram of the control system opening outer loop of the height control system of the UAV given in Figure 2. Results of the computer simulation can be seen in Figure 8. From Figure 8 it is evident that frequency domain dynamic performances are as follows:

$$G_m = \infty, \quad P_m = 66.6^0$$  (22)

It is easy to determine that dynamic performances fit frequency domain requirements given in Table 1.

Fig. 8 Open Loop System Bode-diagram.

At this point we can finish our preliminary design task.

4.3. Analysis of the Disturbance Rejection Ability. Preliminary design of the automatic flight control system must be followed by analysis of the disturbance rejection ability of the control system. Figure 2
shows vertical gust $w_g(t)$ acting as environmental disturbance affecting motion of the UAV. It is evident that disturbance $w_g(t)$ depends on many circumstances, and conditions. In this paper not going into deep detail let us suppose to have following initial data and conditions [20]:

$$H = 100 \text{ m} \approx 328,084 \text{ feet};$$

$$U_0 = 25 \text{ m/s} = 90 \text{ km/h}$$

$$0,45 \text{ m/s} \leq \sigma_w \leq 1,8 \text{ m/s}, \quad L_w = 580 \text{ m}, \quad \sigma_w \text{ r.m.s. is gust velocity [m/s], and finally,} \quad L_w \text{ is the integral scale of the atmospheric turbulence [m].}$$

The minimum value of $\sigma_w$ represents NASA parameter for the vertical gust speed at light wind, while maximum value of $\sigma_w$ is defined for moderate the vertical component of the wind [20]. Time histories of the vertical speed component for minimum, and maximum of $\sigma_w$ can be seen in Figure 9.

Controller synthesis using LQR method is performed in Section 4.2.

System parameters for given heuristically set weighting parameters were derived to be as follows:

$$K_2 = [0,7201 \quad 1] ; \quad Y_s = 0,7201 ; \quad Y_c = 1$$

Disturbance rejection ability of the closed loop automatic flight control system can be analyzed using block diagram given in Figure 2. Suppose straight equilibrium flight of the UAV when random turbulence acts on it, while reference signal of the closed loop control system is zero, $H_r(t) = 0$. In this particular case closed loop transfer function of the automatic flight control system is as follows:

$$W_d(s) = \frac{H(s)}{w_g(s)}\bigg|_{H_r(s)=0} = \frac{2s + 4,6005}{2s^2 + 4,6005s + 5}$$

From equation (25) change of the height of the flight easily can be derived as:

$$H(s) = W_d(s)w_g(s)$$

Using equation (26) closed loop control system was analyzed for disturbance rejection. Results of the computer simulation for light and moderate wind weather conditions can be seen in Figure 10.

![Fig. 10 Altitude Time Histories in the Turbulent Air](image)

Fig. 10 Altitude Time Histories in the Turbulent Air

However, from automatic control theory it is known that unwanted effects from external disturbances affecting motion of the UAV can be reduced, or removed totally using PI-controller in the feedforward path of the automatic flight control system. Let transfer function of the series compensator is as follows:

$$Y_c = 1 + \frac{1}{s}$$

Figure 10 shows that closed loop automatic flight control system is unable to maintain altitude constant, i.e. random turbulent air moves UAV from its equilibrium height for given static error (see Figure 10).

For the moderate wind static error can be found as:

$$\Delta H(t) = H_r(t) - H(t) = -H(t)$$

Absolute value of the steady-state error is

$$|\Delta H(t)| \approx 0,7 \text{ m}$$
which is 0.7% of the equilibrium height of $H_0 = 100\, m$. It is easily can be derived that this inaccuracy means high quality although automatic flight control system uses simple series P-controller. Using transfer function of equation (27) transfer function of the closed loop control system can be derived as:

$$W_d(s) = \frac{H(s)}{w_e(s)} \bigg|_{H_e(s)=0} = \frac{2s^2 + 46005s}{2s^3 + 46005s^2 + 5s + 5}$$

(30)

Applying series controller defined by equation (29) in the feedforward path of the closed loop automatic flight control system given in Figure 2 the closed loop control system was tested for disturbance rejection ability. Results of the computer simulation for light and moderate wind weather conditions can be seen in Figure 11.

$$H(t) = \lim_{s \to 0} H(s) = \lim_{s \to 0} w_e(s) W_d(s) = 0$$

(31)

Figure 11 shows that weather conditions affect basically the response from external disturbance of the vertical wind gust. Figures 12 demonstrate responses of the closed loop control system.

Figure 12.1 shows responses for light wind weather conditions, and Figure 12.2 shows response from the automatic flight control system for moderate wind meteorological conditions. Dealing with problems of the analysis of the disturbance rejection ability it is worth to mention that we have to re-arrange analysis of the reference signal tracking ability of the closed loop height control system having PI-controller. We have just kept in mind disadvantages of the integral compensation, regarding automatic control theory. This theory deals in deep details with filtering problems of the closed loop control systems.

Final outcome from here is to apply PID-controller eliminating partly, of fully disadvantages of the integral compensation.

4.4. Transient Response Analysis of the Height Control System. Height control system of the hypothetical UAV having series controller defined by equation (27) was tested both in time, and in frequency domains. Result of the computer simulation can be seen in Figures 13.
Figure 13.1 demonstrates behavior of the closed loop control system in the time domain. Step responses show that applying PI-controller dynamic performances provided with the simple P-controller are going worse, i.e. steady-state response time increased to that of $t_{ss} \approx 8.7$ sec achieving large value of the overshoot. In frequency domain it can be stated that phase margin goes worse having value of $\approx 22.8$ deg. Evaluating these results both form reference tracking and disturbance rejection it is easy can be determined that control engineers must agree in compromises allowing satisfaction of all pre-defined dynamic performances.

Disadvantages of application of the PI-controllers can be omitted using PID-controllers. In our example we have considered following transfer function found heuristically of the series controller:

$$Y_c = 1 + \frac{1}{s} + 0.25s$$  \hspace{1cm} (32)

Using transfer function defined by equation (32) height control system was tested. Results of the computer simulation are summarized in Figures 14.

Figure 14.1 demonstrates responses from all closed loop control systems having P-, PI, and PID-controllers investigated before in previous sections. Results of the time domain analysis of the height control system show that PID - controller improves dynamic performances both in time and in frequency domains, i.e. steady-state response time is decreased, and phase margin is increased (see Figure 14.2). From results of the computer simulation it is obvious that further investigation of the PID-controller tuning is required.

5. SUMMARY AND CONCLUSIONS

This paper deals with computer aided preliminary design of the closed loop control systems, taking for the example the height control system of the hypothetical UAV. Main equations of the LQR design procedure were summarized to find optimal static state feedback gain matrix. The static controller provides for the closed loop control system all
time and frequency domain dynamic performances. Note that decrease of the gain of $Y_{s_1} = 1.1565$ to that of $Y_{s_1} = 0.7201$ means that we reduced control energy, which is very important from other aspects of the design of the UAV systems.

Analysis of the disturbance rejection ability showed that PI-controller is needed to reduce unwanted effects from external disturbances. It was proved that PI-controller minimizing static error of the height control system to its zero value. However, regarding results of the computer simulation both time and frequency domain dynamic performances become worse. For this reason a simple PI-controller is rarely used in automatic flight control systems. Instead of that PID-controllers are used to improve a complex set of dynamic performances. There are many possible solutions applying PID static controllers in autopilots, such as MP2000, or MP2028, which are available on the market for sale. Today in many military UAV systems M2028 is used as on-board autopilot.

REFERENCES

1. Csáki, F., Korszerű szabályozáselmélet, Akadémiai Kiadó, Budapest, 1970;
2. Csáki, F., Korszerű szabályozáselmélet, Akadémiai Kiadó, Budapest, 1970;
15. MIL-F-8785C, Notice 2, Flying Qualities of Pilot Aircraft, 1996;