A METHOD FOR PERIODICAL PHENOMENA ANALYSIS

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***Abstract**: This paper is proposing a statistical method, useful for analyzing periodical phenomena whose equations are impossible to be solved analytically. The input data consist in a collection of waveforms, (numerically determined), named further database. Least square method based algorithms, presented in the first part of the paper, are applied to this database. First, a trigonometric regression algorithm will find the approximating Fourier coefficients.*

Finally a multiple regression algorithm for finding a polynomial type function is introduced. Its input data are the Fourier coefficients, the Fourier analysis results, or whatever collection of experimental data corresponding to a set of variables. The final part of the paper is dedicated to an example, illustrative for these.

***Keywords**: algorithms, approximating functions, Fourier analysis, trigonometric regression, multiple regression.*

1. INTRODUCTION

In technical matters (specially in electrotechnics and electronics) there are a lot of periodical phenomena described by equations whose analytical solutions are impossible to find (i.e. transcendental ones). When facing such situations, the only way is to solve these equations numerically. But this procedure returns neither analytical expressions of the quantities, nor designing relations.

The goal of this paper is to propose a method to find approximating functions that describe phenomena like the above-mentioned ones.

The first step is to build representative samples for the ranges of the variables that influence the analyzed phenomena and to perform all the actions (solve numerically the equations, calculate the waveforms points, and so on). It results in this way a collection of tabled functions that are the input data of the method below-described.

To synthesize the information regarding a tabled function, it is necessary its approximation using a continuous function (named model function), $f = f(\mathbf{x}, a_0, a_1, ..., a_n)$ depending on variables vector $\mathbf{x}$ and adjustable parameters a_j, $j = \overline{0, n}$ (named model parameters).

The model function establishment is not a simple (trivial) problem and it have to be based on a rigorous argumentation. Generally speaking, there are two possibilities in choosing the model function:

- To choose it from a convenient class of function (as polynomials, trigonometric, …), offering in this way simplicity and efficiency in further processing;
- The model function derives from a theory,
- where the model parameters have a well-established significance.

In this paper two least squares method based algorithms will be presented:

- A trigonometric regression algorithm, useful to find approximating Fourier series attached to periodical phenomena;
- A multiple regression algorithm, useful to find polynomial functions depending on *n* variables, offering the possibility of choosing the polynomial degree for each variable.

2. TRIGONOMETRIC REGRESSION

Representing the regression curve by a Fourier series base this algorithm.

This procedure is useful in analyzing periodical phenomena, knowing the frequency value.

It will be the fundamental frequency. It results in this way the model function:

$$x(t)=\sum_{n=0}^{\infty}\left(C_n\cos n\omega t+S_n\sin n\omega t\right) \qquad (01)$$

The least squares method impose to minimize the deviation function:

$$F(C_0, C_1, C_2, \ldots S_1, S_2, \ldots)= =\sum_{i=1}^{\infty}\left(X_i-\sum_{j=0}^{\infty}\left(C_j\cos j\omega t_i+S_j\sin j\omega t_i\right)\right)^2 \qquad (02)$$

where (t_i , X_i) are the tabled values, and ω the pulsation corresponding to the fundamental frequency. By nullifying the deviation function first partial derivatives results a linear system of equations, the unknowns being the Fourier coefficients.

The system matrix results as it follows:

$$A_1=\left[\sum_{i=1}^{m}\cos(k\omega t_i)\cos(j\omega t_i)\right]_{k=\overline{0,n};\, j=\overline{0,n}} \qquad (03)$$

$$A_2=\left[\sum_{i=1}^{m}\cos(k\omega t_i)\sin(j\omega t_i)\right]_{k=\overline{0,n};\, j=\overline{1,n}} \qquad (04)$$

$$A_3=\left[\sum_{i=1}^{m}\sin(k\omega t_i)\cos(j\omega t_i)\right]_{k=\overline{1,n};\, j=\overline{0,n}} \qquad (05)$$

$$A_4=\left[\sum_{i=1}^{m}\sin(k\omega t_i)\sin(j\omega t_i)\right]_{k=\overline{1,n};\, j=\overline{1,n}} \qquad (06)$$

$$A_5=\mathrm{augment}(A_1, A_2) \qquad (07)$$

$$A=\mathrm{stack}(A_5, A_6) \qquad (08)$$

The constant terms vector results as it follows:

$$B_1=\left[\sum_{i=1}^{m}x_i\cos(k\omega t_i)\right]_{k=\overline{0,n}} \qquad (09)$$

$$B_2=\left[\sum_{i=1}^{m}x_i\sin(k\omega t_i)\right]_{k=\overline{1,n}} \qquad (10)$$

$$B=\mathrm{stack}(B_1, B_2) \qquad (11)$$

The Fourier coefficients are:

$$\left[C_0\, C_1 \ldots C_n\, S_1 \ldots S_n\right]^T = A^{-1}\cdot B \qquad (12)$$

A MathCAD function was written to perform the formulas (3) ... (12). Its input data consists in a two-columned matrix (two vectors) containing the waveforms points, $\left[t_i \quad x_i\right]_{i=\overline{1,m}}$, and n, the desired number of harmonics.

The function performs additionally the harmonic Fourier series coefficients, and phase shifts starting from the quantities returned by (12).

3. THE MULTIPLE REGRESSION

In this chapter a multi-variable polynomial type approximation function will be build (multiple regression).

$$y(x_1, x_2, \ldots, x_n)= =\sum_{i_1=0}^{p_1}\sum_{i_2=0}^{p_2}\ldots\sum_{i_n=0}^{p_n}a_{i_1i_2\ldots i_n}x_1^{i_1}x_2^{i_2}\ldots x_n^{i_n} \qquad (13)$$

where x_1, x_2, …, x_n are the function variables, p_1, p_2, …, p_n are the polynomials degrees according to each variable respectively and $a_{i_1i_2\ldots i_n}$ the coefficients.

The method consists in a sequence of performing of a polynomial regression algorithm for a single-variable model function:

$$f(z, a_0, a_1, \ldots, a_d)= =a_dz^d+a_{d-1}z^{d-1}+\ldots+a_1z+a_0 \qquad (14)$$

The method consists in a sequence of performing of a polynomial regression algorithm for a single-variable model function:

$$F(a_0, a_1, \ldots, a_d)= =\sum_{i=1}^{m}\left(a_dz_i^d+\ldots+a_1z_i+a_0-y_i\right)^2 \qquad (15)$$

The coefficients result as it follows:

$$[a_k]_{k=\overline{0,d}} = \left(\left[\sum_{i=1}^{m} z_i^{k+j} \right]_{\substack{k=\overline{0,d} \\ j=\overline{0,d}}} \right)^{-1} \cdot \left(\left[\sum_{i=1}^{m} y_i z_i^k \right]_{k=\overline{0,d}} \right) \quad (16)$$

where (z_i, y_i) are the tabled values of the function $y(z)$.

It is easy to observe the algorithmic structure of (16), so it was implemented as a MathCAD function. Its input data is the matrix (two vectors) containing the tabled values of the function and d, the polynomial degree.

The least squares method can be applied in the same way for a multi-variable function, but finding an algorithm to do it for whatever number of variables and corresponding degrees becomes a difficult task.

For this reason a method in this direction will be the subject treated in the next part of the paper. The task is to find the coefficients $a_{i_1 i_2 \ldots i_n}$ from (13) and the method will consist in a sequence applying of the single-variable polynomial regression algorithm, described by (16).

The algorithm input data is a matrix containing the function and variables tabled values and a vector containing the desired polynomials degrees.

The above-mentioned matrix has to be arranged as the points of a hypercube, the last column containing the function tabled values.

The algorithm consists in the following steps:

- Polynomial coefficients corresponding to the last variable are determined, considering the other ones as fixed. It results in this way a collection of coefficients, associated to each value of the fixed variables (a vector $a_0^{i_1 i_2 \ldots}$ - constant terms, a vector $a_1^{i_1 i_2 \ldots}$ - first powers coefficients and so on, where i_1, i_2, ... are the indexes corresponding to each fixed variables).
- These vectors will be considered as dependent variables (functions) for the next step, the independent one being the next variable of the tabled function. It results in this way a new set of polynomial coefficients.
- This procedure is sequenced till exhausting the tabled function variables.

3.1. AN EXAMPLE

A 3 variables function will be approximated: $\beta(\alpha, \varepsilon, \varphi)$.

This function is supposed to be previously tabled, as shown in table 1, and $\alpha, \varepsilon, \varphi$ tabled values are:

$\alpha = \alpha_0, \alpha_1, \ldots \alpha_v ; \varepsilon = \varepsilon_0, \varepsilon_1, \ldots \varepsilon_t ;$

$\varphi = \varphi_0, \varphi_1, \ldots \varphi_u$

The approximations will be performed with polynomials having the degrees n, p and q, corresponding to $\alpha, \varepsilon, \varphi$ respectively.

The first step will provide a set of functions

$$\beta_{\varphi_i, \varepsilon_j}(\alpha) = a_0^{ij} + a_1^{ij}\alpha + \ldots + a_n^{ij}\alpha^n ,$$

corresponding to each value ε_j and φ_i of variables ε, and φ, respectively, in this step being determined the coefficients a_k^{ij}, $i = \overline{0,u}$; $j = \overline{0,t}$; $k = \overline{0,n}$.

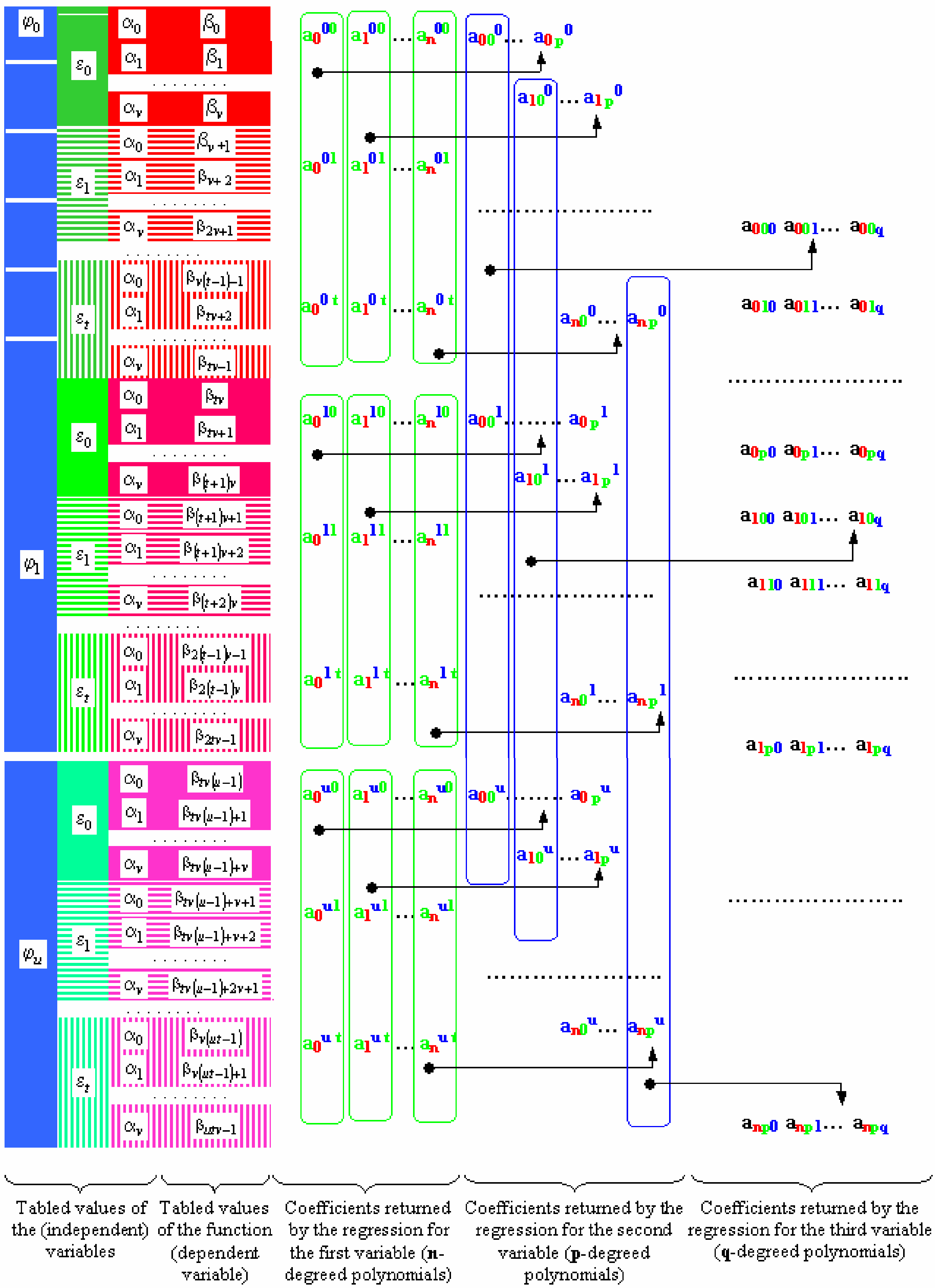
The second regression will be applied to previously determined coefficients a_k^{ij}, considering the second variable (ε) as independent one, obtaining in this way a new set of functions (alike the ones above-mentioned), corresponding to each φ value, φ_s, $s = \overline{0,u}$.

Using the coefficients a_{wz}^s, $w = \overline{0,n}$; $z = \overline{0,p}$ determined in this step, the set of functions (two-variables) is written as it follows:

$$\begin{aligned} \beta_{\varphi_s}(\varepsilon, \alpha) &= a_{00}^s + a_{01}^s\varepsilon + \ldots + a_{0p}^s\varepsilon^p + \\ &+ \left(a_{10}^s + a_{11}^s\varepsilon + \ldots + a_{1p}^s\varepsilon^p\right)\alpha + \ldots \\ &\ldots + \left(a_{n0}^s + a_{n1}^s\varepsilon + \ldots + a_{np}^s\varepsilon^p\right)\alpha^n \end{aligned} \quad (17)$$

The last regression will be applied to a_{wz}^s, considering φ as independent variable, obtaining in this way the coefficients $a_{i_1 i_2 \ldots i_n}$.

Table 1. Processing diagram for a three variables function



The (final) regression function is:

$$\beta(\varphi,\varepsilon,\alpha)=$$
$$=\begin{bmatrix}\left(a_{000}+a_{001}\varphi+\ldots+a_{00q}\varphi^{q}\right)\varepsilon^{0}+\\+\left(a_{010}+a_{011}\varphi+\ldots+a_{01q}\varphi^{q}\right)\varepsilon+\ldots\\+\left(a_{0p0}+a_{0p1}\varphi+\ldots+a_{0pq}\varphi^{q}\right)\varepsilon^{p}\end{bmatrix}\alpha^{0}+$$
$$+\begin{bmatrix}\left(a_{100}+a_{101}\varphi+\ldots+a_{10q}\varphi^{q}\right)\varepsilon^{0}+\\+\left(a_{110}+a_{111}\varphi+\ldots+a_{11q}\varphi^{q}\right)\varepsilon+\ldots\\+\left(a_{1p0}+a_{1p1}\varphi+a_{1pq}\varphi^{q}+\ldots\right)\varepsilon^{p}\end{bmatrix}\alpha+$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+\begin{bmatrix}\left(a_{n00}+a_{n01}\varphi+\ldots+a_{n0q}\varphi^{q}\right)\varepsilon^{0}+\\+\left(a_{n10}+a_{n11}\varphi+\ldots+a_{n1q}\varphi^{q}\right)\varepsilon+\ldots\\+\left(a_{np0}+a_{np1}\varphi+\ldots+a_{npq}\varphi^{q}\right)\varepsilon^{p}\end{bmatrix}\alpha^{n}$$

4. CONCLUSION

The authors used these algorithms MathCAD versions in many applications, one of them being the analysis of the periodical a.c.-switching mode of a single-phased transformer [3, 4].

In this chapter the main results obtained in RL load with leakage diode case will be presented.

The first step was to fix a representative sample for the load (8 phase shifts φ between 0.1^0 and 89.9^0) and the switch firing angles range (9 values for α_0 between 0^0 and 179^0; the switch was considered thyristor type).

The second step was the steady state waveforms numerical determination, resulting in this way the input database mentioned in section 1.

The trigonometric regression algorithm was applied to all the resulted signals.

In figure1 is shown a comparison between a numerically determinate signal and its corresponding harmonic Fourier series.

Fourier coefficients were used to perform the Fourier analysis whose result was the power consumptions determination.

This was the input data for the multiple polynomial regression algorithms.

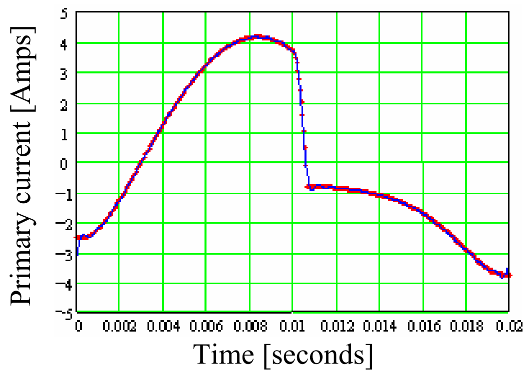


Fig. 1 The primary current:
Numerically determined (points)
Approximated (full line)

In figure 2 is shown the primary apparent power surface (a function of φ and α_0) and in figure 3 the errors between the primary power consumption returned by the Fourier analysis and the regression function.

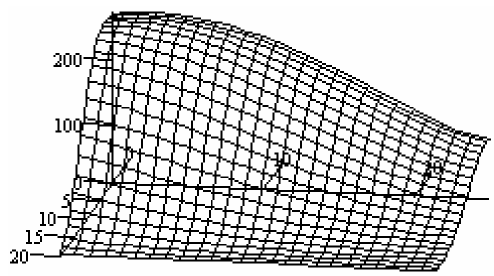


Fig. 2 The primary apparent power as a function of load phase shift, φ,
and firing angle, α_0

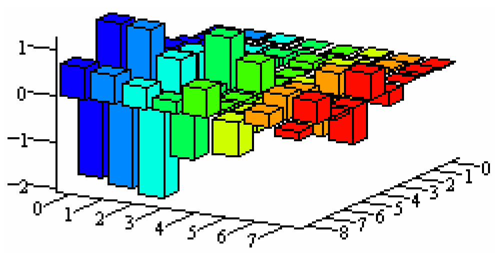
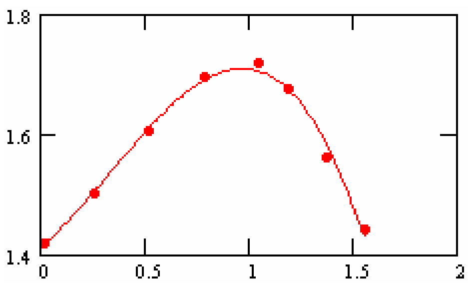


Fig. 3 The errors between power consumptions returned by the Fourier analysis and the ones obtained by using its polynomial approximating function for the diode case

In figure 4 is shown the inverse duty factor of the transformer (the ratio between the transformer apparent power and the active power in the load circuit) as a function of φ, for $\alpha_0 = 0$ (the diode case).

The function represented in figure 4 can be used in designing transformers working in such regimes, to determine its apparent power, which is the main input data of this process.



Load phase shift [radians]

Fig. 4 The transformer inverse duty factor as a function of load phase shift,

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