MODEL PREDICTIVE CONTROL APPLIED IN UAV FLIGHT PATH TRACKING MISSIONS

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Abstract: *The UAV flight path design often represents the biggest problem ~~of~~ when it comes to the optimization of common meaning, like the minimum energy problem, maneuvers executed the fastest way. The problem of flight path optimization requires some predictions to eliminate or to minimize the probability of collisions in all possible situations, such as the collision of two or more UAVs, a UAV and a manned aircraft, UAVs and static objects, UAVs and flying birds etc. The Model Predictive Control (MPC) is one of the best methods to control selected UAV paths. The goal of the author is to highlight the mathematical backgrounds concerning the MPC problem formulation, based on the receding horizon problem (RHP), and on the Laguerre functions method.*

Keywords: *UAV flight path design, UAV optimal trajectory, receding horizon problem, Laguerre functions.*

1. INTRODUCTION

Unmanned aerial vehicles (UAV) are going to be used in wide variety. The recent history of UAV applications is awash in informative articles describing brand-new UAV applications like urban drone taxis, drones used for sightseeing, rideable by man (hover bike) drones used by police forces and military squads. UAV applications in urban areas are generating a set of new challenges designers must answer and solve.

As a first challenge, if the UAV is integrated into national airspace management, is to be able to design a flight path ensuring flight safety at the same (or higher) level as compared to manned aviation. Due to the limited amount of electrical energy stored into the batteries, the flight path of the UAV must be optimized in a few ways; for instance, the flight time must be maximized whilst the energy required for a given maneuver has to be minimized and the entire flight must be planned.

The model predictive control may provide solutions for these problems related to UAV flight path design famous for optimization with constraints. The optimal control law being constrained will steer UAV from the initial equilibrium set point to the next equilibrium flight regime minimizing a pre-chosen integral performance index, say, the closed loop cost function.

Basic idea of the MPC can be formulated using receding horizon problem, which is a common and widely used formulation in control theory. The competing method to the receding horizon problem is the optimal control formulation, which has many advantages to those problems solved using RHP.

2. RELEVANT REFERENCES AND PRELIMINARIES

The pioneering work of Löfberg deals with MPC design problem formulation and with its solution for constrained systems using LMI method [1].

The early works of Seeborg et al. formulates basic idea of the receding horizon problem, and many applications are available to highlight importance of the topic of MPC control system design [2, 13]. In [3] preliminary design of the UAV longitudinal controller was introduced, in [5] the important topic of the redundancy of the UAV is investigated. The UAV spatial motion automation based upon model predictive control in different flight scenarios is thoroughly examined in [4]. The evolution of the UAVs is deeply analyzed in [7], and its military application in integrated air defense systems is examined in [6]. In [8] the MPC is introduced for vehicle control, for traction control, for power systems, and for product planning as well. This paper will demonstrate a numerical example based on [9], and [10]. The UAV continuous dynamic system MPC design will be conducted with pre-chosen design parameters [11, 12]. UAV and UAS innovative solutions are deeply examined in [17], and UAV launch system is discussed in [18].

3. FUNDAMENTALS OF THE MODEL PREDICTIVE CONTROL

The general idea of the MPC and its main objectives have been formulated by Seeborg et al. as follows [2]:

– prevent violations of control input and predicted output constraints;

– drive process output to their optimal set points maintaining remaining process outputs within specified ranges;

– prevent aggressive changes of the input variables;

– control maximum number of the process variables when the sensor or the actuator are not available in the closed loop control systems.

The MPC system is very suitable for solution of the constrained MIMO control problems, which is typical for many UAV types. The MPC system block diagram can be seen in Fig. 1. [4].

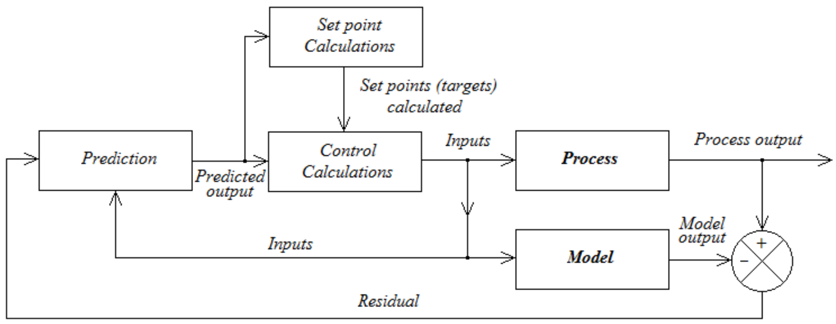


FIG. 1. Block Diagram of the MPC.

The process model is used to predict the current values of the output variables. The differences called residuals measured between process outputs and model outputs used as feedback signal to the prediction block, which is also subjected to the input signals. At every sampling time, two types of calculations, namely the set-point calculations and control calculations are performed. During either calculation inequality constraints like lower and/or upper limits are set upon output variables.

Control calculations lead to inputs subjected to both process and model paths. The set-points (targets) are calculated leaning on well-known optimization criteria of the cost minimization. Worth to mention, that in industrial applications targets (set-points) are calculated using economic optimization procedures of the production rate maximization, or, as a rule of the profit maximization.

The optimum values of the targets will change due to varying process conditions (e.g. noises, parameter variations, system uncertainties, changes in inequality bounds, etc.). The constraints of the output variables change due to varying process conditions, equipment and instrumentation. In MPC systems set-points (targets) are re-calculated each time when the control calculations are performed. All two types of the calculations are based upon current measurements and the predicted (future) value of the process outputs.

The main goal of the MPC control calculations is to determine the control inputs required to drive predicted process outputs to its optimal targets.

4. MATHEMATICAL FORMULATION OF THE RECEDING HORIZON CONCEPT

Behind the basic concept of the MPC is the idea of the receding horizon [1, 2, 4, 8, 11, 12, 13], which is illustrated in Fig. 2.

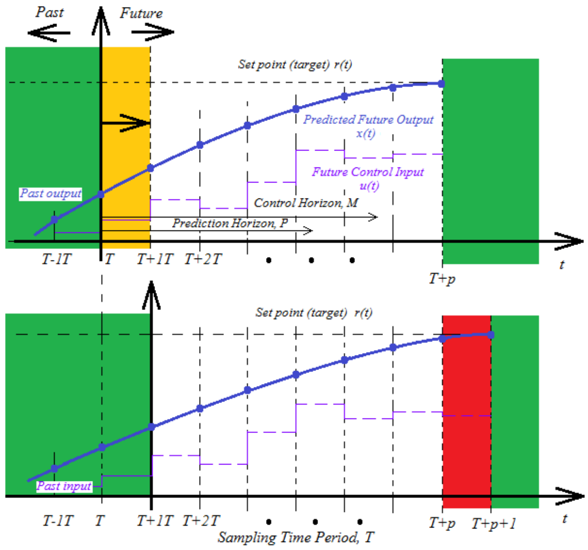


FIG. 2. Basic Concept of the MPC.

The MPC problem is mostly formulated for discrete time systems. At any initial discrete time of the sampling, say, $t=T$, the predicted *process* output is calculated using the internal *model* response. This calculation is performed for the entire range of the of the prediction horizon of p. Leaning on process predicted outputs and model outputs at time '$t=T+p$' control effort needed to minimize system error (residuals) is calculated to drive process to follow the optimal reference trajectory. First step in this concept is: at time '$t=T+1T$' the calculated control input is executed. At the same time, the process output is measured, and compared with the internal model outputs. Based upon the residuals (errors) the new predicted future output is calculated for the new horizon of '$t=T+2T$', till the discrete time of '$t=T+p+1$' on the horizon, p.

This means that prediction horizon keeps to be shifted, at every time, p seconds ahead of the current time, t.

In spite of being interested in discrete time models, continuous time MPC models using orthonormal functions are also interested due to their reduced required computer power needed for calculations [4].

This article will lean upon continuous time models used to calculate process outputs and desired optimal control trajectory.

However, execution of the algorithm proposed will use discrete time settings that for a pre-defined time step the control effort is executed before the next optimization step is made [2, 4, 8, 11, 12, 13].

5. MPC OPTIMAL CONTROL PROBLEM FORMULATION

The MPC problem is formulated for the multivariable dynamic system. The process (plant) behavior is described by the following nonlinear equation [2, 4]:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \tag{1}$$

The model outputs are calculated up to the horizon time $t=p$, which represents a terminal point of the calculations. Let us set the control objective as: minimize a cost function of the form as given below:

$$\boldsymbol{V}(\boldsymbol{x}, \boldsymbol{u}, t) = \int_o^p \boldsymbol{l}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)dt + \boldsymbol{F}\big(\boldsymbol{x}(p)\big)\,, \tag{2}$$

where $\boldsymbol{l}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \geq 0$, and $\boldsymbol{F}\big(\boldsymbol{x}(p)\big)$ is the terminal state weighting at $t=T+p$, and, control input $u(t)$ is subjected to some constraint of $\boldsymbol{u}(t) \in \boldsymbol{U}$. The formulation of the control problem based upon Eqs (1) and (2) is a very general one, and, proper choice of functions $\boldsymbol{f}$, $\boldsymbol{F}$ and $\boldsymbol{l}$ can lead to very common representations of sensible problems.

Solution of the (2) cost function minimization problem requires solution of the partial differential equation formulated below:

$$\frac{\partial}{\partial t}\boldsymbol{V}^0(\boldsymbol{x}, t) = min_{u \in U}\boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u}, \frac{\partial}{\partial t}\boldsymbol{V}^0(\boldsymbol{x}, t)) \tag{3}$$

In Eq (3): $\boldsymbol{H}(\boldsymbol{x}, \boldsymbol{u}, \lambda) = \boldsymbol{l}(\boldsymbol{x}, \boldsymbol{u}) + \lambda \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u})$ is the Hamiltonian function with the boundary condition of $\boldsymbol{V}(\boldsymbol{x}, \boldsymbol{p}) = \boldsymbol{F}(\boldsymbol{x}(p))$, and, λ is the Lagrange multiplier. Eq (3) represents the well-known Hamilton-Bellman-Jacobi equation. So as to be able to solve Eq (3) some assumptions needed and must be introduced. Further we will assume that the plant (process) is a linear one so that function $\boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t)$ will gain special form of:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t), \tag{4}$$

which represents the MIMO time varying system state equation [2, 4]. Functions $\boldsymbol{l}$ and $\boldsymbol{F}$ will have quadratic form as follows below [2, 4, 13]:

$$\left.\begin{aligned} \boldsymbol{l}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) &= \boldsymbol{x}^T(t)\boldsymbol{Q}(t)\boldsymbol{x}(t) + \boldsymbol{u}^T(t)\boldsymbol{R}(t)\boldsymbol{u}(t) \\ \boldsymbol{F}\big(\boldsymbol{x}(p)\big) &= \boldsymbol{x}^T(p)\boldsymbol{S}(t)\boldsymbol{x}(p) \end{aligned}\right\} \tag{5}$$

In eq (5): $\boldsymbol{Q}(t) \geq 0$, $\boldsymbol{S}(t) \geq 0$, $\boldsymbol{R}(t) > 0$ are square weighting matrices. These conditions will drive to a special case when the Hamilton-Bellman-Jacobi equation simplifies to the ordinary differential equation (ODE) of Ricatti.

The Hamilton-Bellman-Jacobi equation also can be solved if to introduce:

$$V^0(x, t) = x^T(t)P(t)x(\mathrm{t})\,,\ \ \boldsymbol{where}\ \ P(t) = P^T(t) \text{ – cost matrix.} \tag{6}$$

Then, the Hamilton-Bellman-Jacobi equation may be rewritten as given below:

$$\left.\begin{aligned} -\dot{\boldsymbol{P}}(t) &= \boldsymbol{P}(t)\boldsymbol{A}(t) + \boldsymbol{A}^T(t)\boldsymbol{P}(t) + \boldsymbol{Q}(t) - \boldsymbol{P}(t)\boldsymbol{B}(t)\boldsymbol{R}^{-1}(t)\boldsymbol{B}^T(t)\boldsymbol{P}(t) \\ \boldsymbol{P}(p) &= \boldsymbol{S} \end{aligned}\right\} \tag{7}$$

Eq (7) can be solved and it leads to a linear time-varying feedback control law:

$$\boldsymbol{u}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t), \tag{8}$$

where $\boldsymbol{F}(t)$ stands for the state feedback gain matrix, depending on $\boldsymbol{P}(t)$, which is the solution of the Ricatti equation (7):

$$\boldsymbol{F}(t) = \boldsymbol{R}^{-1}(t)\boldsymbol{B}^{T}(t)\boldsymbol{P}(t) \tag{9}$$

Difficulties related to solution of Eq (7) can be eliminated using candidate method of using orthonormal functions for continuous time MPC design.

6. CONTINUOUS TIME MPC DESIGN BASED ON ORTHONORMAL FUNCTIONS

The design technique based on orthonormal functions and presented by [2, 4] can be used as an alternative one to the classical receding horizon control solution requiring solution of the Ricatti ODE. Future prediction is calculated in an analytical form, the required control trajectory is calculated using a pre-chosen set of orthonormal basis functions. This technique basically has been developed for the continuous time systems, however, it can be extended to the discrete time systems, and, expected to be extended to nonlinear systems, too. In [4] Laguerre orthonormal functions are used and proposed to reduce complexity of the performance specification process. The uniqueness of this procedure is that problem of finding optimal control signals required is turned into the finding a set of coefficients for the Laguerre model. This technique reduces the number of required parameters in the calculations, and, has important advantage when it is used in on-line environment.

6.1 Defining control trajectory. It is well-known that any arbitrary function *f(t)* can be expanded into formal expansion analogue to that of the Fourier expansion. Any arbitrary function *f(t)* can be expressed in the following series expansion:

$$f(t) = \sum_{i=1}^{\infty} \xi_i \, l_i(t) \,, \; i=1,2,3 \ldots \tag{10}$$

where ξ_i are coefficients of the orthonormal functions $l_i(t)$ satisfying following conditions:

$$\int_0^{\infty} l_i^2(t)\, dt = 1; \; \int_0^{\infty} l_i(t)\, l_j(t) dt = 0; \quad \forall i \neq j \tag{11}$$

Secondly, assuming that *f(t)* represents a piece-wise continuous function satisfying

$$\int_0^{\infty} f^2(t)\, dt < \infty, \tag{12}$$

then for any $\varepsilon > 0$ over the time range of $0 \leq t \leq \infty$ there is an existing finite integer of *N* such that for all $k \geq N$

$$\int_0^{\infty} (f(t) - \sum_{i=1}^{k} \xi_i \, l_i(t))^2 \, dt < \varepsilon \tag{13}$$

In other words, the truncated expansion of $\sum_{i=1}^{N} \xi_i \, l_i(t)$ is used to closely approximate any arbitrary function *f(t)*. One famous set of the orthonormal functions used frequently is a set of the Laguerre functions important to engineers because of simple Laplace transforms of the $l_i(t)$, i.e.:

$$\int_0^{\infty} l_i(t) e^{-st} \, dt = \sqrt{2p} \, \frac{(s-p)^{i-1}}{(s+p)^{i}}, \tag{14}$$

In Eq (14) $p > 0$ and called scaling factor. From Eq (14) a differential equation satisfying Laguerre functions can be derived. Let:

$$\left.\begin{aligned} \boldsymbol{L}(t) &= [l_1(t) \quad l_2(t) \quad \cdots \quad l_N(t)]^T \\ \boldsymbol{L}(0) &= \sqrt{2p}[1 \quad 1 \quad \cdots \quad 1]^T \end{aligned}\right\} \tag{15}$$

The Laguerre functions will satisfy the differential equation given below:

$$\dot{\boldsymbol{L}}(t) = \boldsymbol{A}_p \boldsymbol{L}(t);\ \boldsymbol{A}_p = \begin{bmatrix} -p & 0 & \cdots & 0 \\ -2p & -p & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ -2p & \cdots & -2p & -p \end{bmatrix} \tag{16}$$

The solution of the differential equation (16) represents the Laguerre functions in the following matrix exponential function:

$$\boldsymbol{L}(t) = e^{\boldsymbol{A}_p t} \boldsymbol{L}(0) \tag{17}$$

For LTI systems when the closed loop control system is the stable one, after the transient response period the control signal calculated for the given set-point will exponentially converge to a given constant. Leaning on this, in the receding horizon control design problem solution the future control input signal calculated for each moving window will be invariant one, in other words, $\dot{\boldsymbol{u}}(t) = 0$ for each horizon window of $T_i \leq t \leq T_i + p$. Easy to see that:

$$\int_{T_i}^{T_i+p} \dot{\boldsymbol{u}}^2(t)dt < \infty \tag{18}$$

The derivative of the future control input signal can be determined using following Laguerre function representing series expansion as given below:

$$\dot{\boldsymbol{u}}(t) = \sum_{i=1}^{\infty} \xi_i\, l_i(t) = \boldsymbol{L}^T(t)\boldsymbol{\eta} \tag{19}$$

In Eq (19): $\boldsymbol{\eta} = [\xi_1 \quad \xi_2 \quad \cdots \quad \xi_N]^T$ column vector of the coefficients of the orthonormal functions $l_i(t)$ used to express expansion of $\dot{\boldsymbol{u}}(t)$.

6.2 Predicted process (plant) output. The dynamic plant to be controlled is assumed to be a MIMO system with control inputs ***u(t)*** of dimension *r*, and outputs ***y(t)*** of the dimension of *q*.

To gain realistic environment in which MPC control problem is being solved, the plant is subjected to external disturbances ***w(t)*** (e.g. atmospheric turbulences, air temperature changes, air density changes, air pressure changes etc.), and measurement process is a noisy one, outputs are disturbed with sensor noises ***n(t)***.

It is supposed that $\dot{\boldsymbol{w}}(t)$ and $\dot{\boldsymbol{n}}(t)$ are continuous time uncorrelated white noise processes with zero means, thus we have [4]:

$$\begin{aligned} &E\left\{\frac{d\boldsymbol{w}(t)}{dt}\right\} = 0\ ;\ E\left\{\frac{d\boldsymbol{n}(t)}{dt}\right\} = 0; \\ &E\left\{\frac{d\boldsymbol{w}(t)}{dt}\frac{d\boldsymbol{w}^T(\tau)}{d\tau}\right\} = W_w \delta(t-\tau);\ E\left\{\frac{d\boldsymbol{n}(t)}{dt}\frac{d\boldsymbol{n}^T(\tau)}{d\tau}\right\} = R_n \delta(t-\tau) \end{aligned} \tag{20}$$

In Eq (20): $E\{\quad\}$ stands for expected value, $\delta(\quad)$ is the Dirac function, W_w and R_n are disturbance and noise intensities, respectively.

So we have the LTI MIMO system model in the following standard matrix form [2, 4]:

$$\left.\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{w}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) + \boldsymbol{n}(t) \end{aligned}\right\} \tag{21}$$

Let us introduce a new variable of the form: $\boldsymbol{z}(t) = \dot{\boldsymbol{x}}(t)$. We have now:

$$\dot{\boldsymbol{z}}(t) = \ddot{\boldsymbol{x}}(t) = \frac{d}{dt}\{\boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{w}(t)\} \tag{22}$$

Using Eq (22) the dynamic system state equation (21) can be re-formulated in the following augmented form:

$$\left.\begin{aligned}\dot{\boldsymbol{z}}(t) = \ddot{\boldsymbol{x}}(t) = \boldsymbol{A}_a\boldsymbol{X}(t) + \boldsymbol{B}_a\dot{\boldsymbol{u}}(t) + \begin{bmatrix}\dot{\boldsymbol{w}}(t)\\ \dot{\boldsymbol{n}}(t)\end{bmatrix}\\ \boldsymbol{y}(t) = \boldsymbol{C}_a\boldsymbol{X}(t)\end{aligned}\right\} \tag{23}$$

In Eq (23):

$$\boldsymbol{X}(t) = \begin{bmatrix}\boldsymbol{z}(t)\\ \boldsymbol{y}(t)\end{bmatrix}\ ;\ \boldsymbol{A}_a = \begin{bmatrix}\boldsymbol{A} & 0\\ \boldsymbol{C} & 0\end{bmatrix}; \boldsymbol{B}_a = \begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{D}\end{bmatrix}; \boldsymbol{C}_a = [0 \quad \boldsymbol{I}]; \tag{24}$$

In Eq (24): $\boldsymbol{I}$ is an identity matrix of dimensions of $q \times q$. Worth to mention that in the augmented MIMO system state equation (23) the control input is the first derivative $\dot{\boldsymbol{u}}(t)$ taken by time from control input $\boldsymbol{u}(t)$, while the system output $\boldsymbol{y}(t)$ remains the same vector, and, the augmented dynamic system represented by Eq (23) is observable and controllable.

Due to special features of the random external and internal disturbances described above the expected effects from them in the future predictions are assumed to be zero. It is supposed that an observer is used to determine plant disturbance $\boldsymbol{w}(t)$ and measurement noise $\boldsymbol{n}(t)$, and their amplitudes are not magnified due to explicit derivations in Eq (23). Regarding this property further considerations of disturbance $\boldsymbol{w}(t)$ and measurement noise $\boldsymbol{n}(t)$ are neglected.

It is assumed that any current time of sampling $t = T_i$ the state variable of the augmented system $\boldsymbol{X}(T_i)$ is available. At any future time $t = T_i + T$ the predicted augmented state variable $\boldsymbol{X}(T_i + T)$, with no expected effects from plant disturbance $\boldsymbol{w}(t)$ and measurement noise $\boldsymbol{n}(t)$ in the future predictions, can be described with following equation [2, 4]:

$$\begin{aligned}&\boldsymbol{X}(T_i + T) = e^{\boldsymbol{A}T_i}\boldsymbol{X}(T_i) + \int_{T_i}^{T_i+T} e^{\boldsymbol{A}(T_i+T-\beta)}\,\boldsymbol{B}\dot{\boldsymbol{u}}(\beta)d\beta = \\ &= e^{\boldsymbol{A}T_i}\boldsymbol{X}(T_i) + \int_0^{T_i} e^{\boldsymbol{A}(T_i-\gamma)}\,\boldsymbol{B}\dot{\boldsymbol{u}}(T_i+\gamma)d\gamma\end{aligned} \tag{25}$$

The projected future control trajectory of $\dot{\boldsymbol{u}}(t)$ can be expressed in the form given below:

$$\dot{\boldsymbol{u}}(t) = [\dot{u}_1(t) \quad \dot{u}_2(t) \quad \dots \quad \dot{u}_r(t)]^T \tag{26}$$

The input matrix of the dynamic MIMO system (21) is as follows:

$$\boldsymbol{B} = [B_1 \quad B_2 \quad \dots \quad B_r] \tag{27}$$

In Eq (27): B_i is the *i*-th column of the input matrix $\boldsymbol{B}$. The *i*-th control signal of Eq (26) $\dot{u}_i(t)$ $(i = 1, 2, 3, \dots, r)$ can be represented using following formula:

$$\dot{u}_i(t) \cong \boldsymbol{L}_i^T(t)\boldsymbol{\eta}_i \tag{28}$$

In Eq (28):

$\boldsymbol{L}_i^T(t) = \left[l_1^i(t) \quad l_2^i(t) \quad \dots \quad l_{N_i}^i(t)\right]$; $\boldsymbol{\eta}_i^T(t) = \left[\eta_1^i(t) \quad \eta_2^i(t) \quad \dots \quad \eta_{N_i}^i(t)\right]$, and N_i is pre-chosen. The predicted augmented state $\boldsymbol{X}(T_i + T)$ at $t = T_i + T$ is as follows:

$$\boldsymbol{X}(T_i+T)=e^{\boldsymbol{A}T_i}\boldsymbol{X}(T_i)+\int_0^{T_i}e^{\boldsymbol{A}(T_i-\gamma)}[B_1L_1^T(\boldsymbol{\eta})\quad B_2L_2^T(\gamma)\quad \dots\quad B_rL_r^T(\gamma)]\,\boldsymbol{\eta}d\gamma \tag{29}$$

In Eq (29) coefficient vector $\boldsymbol{\eta}^T=[\eta_1\quad \eta_2\quad \cdots\quad \eta_r]$ has dimension of $\sum_{i=1}^{r}N_i$. The predicted output of the plant can be determined as:

$$\boldsymbol{y}(T_i+T)=\boldsymbol{CX}(T_i+T) \tag{30}$$

Solution of Eq (29) represents the convolution operation requiring solution of $(n+q)\times\sum_{i=1}^{r}N_i$ integral equation, which means a huge computation load needed for calculations. To minimize that load integral equations can be solved numerically using their finite sum approximations. An analytical solution can be found to the convolution integral corresponding to the i-th input [4]:

$$\boldsymbol{I_{int}}(T_i)^i=\int_0^{T_i}e^{\boldsymbol{A}(T_i-\gamma)}B_iL_i^T(\gamma)\,d\gamma \tag{31}$$

In Eq (31) $\boldsymbol{I_{int}}(T_i)^i$ represents a matrix with dimensions of $(n+q)\times N_i$. Substituting eq (31) into Eq (29) shows that the future prediction of the plant output trajectory can be expressed in terms of convolution integral of (31), if to suppose that $1\le i\le r$. In this case matrix $\boldsymbol{I_{int}}(T_i)^i$ can be derived as follows:

$$\boldsymbol{AI_{int}}(T_i)-\boldsymbol{I_{int}}(T_i)\boldsymbol{A}_p^T=-\boldsymbol{BL}^T(T_i)+e^{\boldsymbol{A}T_i}\boldsymbol{BL}^T(0) \tag{32}$$

Obtaining matrices of $\boldsymbol{I_{int}}(T_i)^i$ for $i=1,2,3,\dots,r$ the future prediction of $\boldsymbol{X}(T_i+T)$ can be determined. Finally, leaning on Eq (30) the predicted plant output also can be calculated.

6.3 Dynamic optimal control of MPC Systems. In MPC of UAVs the cost function is applied for optimization (minimization). Supposing that future set-points $\boldsymbol{r}(T_i+T)=[r_1(T_i+T)\quad r_2(T_i+T)\quad \dots\quad r_q(T_i+T)]$ are available for prediction horizon of $0\le T_i\le T+p$. The common goal of the MPC is to find optimal control input vector driving the predicted plant output $\boldsymbol{x}(T_i+T)$ as close as possible (in the least square meaning) to the future set point of the $\boldsymbol{r}(T_i+T)$, i.e. error $\boldsymbol{e}(T_i+T)=\boldsymbol{r}(T_i+T)-\boldsymbol{y}(T_i+T)$ measured between predicted output and set point must be minimized. In this case, the integral performance index used for optimal control law synthesis can be derived as follows [2, 4]:

$$J=\int_0^{T_p}\{[r(T_i+T)-y(T_i+T)]^T\boldsymbol{Q}[r(T_i+T)-y(T_i+T)]+\dot{u}^T(T)\boldsymbol{R}u(T)\}\rightarrow Min \tag{33}$$

In Eq (33) $\boldsymbol{Q}$ and $\boldsymbol{R}$ are symmetric weighting matrices with $\boldsymbol{Q}>0$ and $\boldsymbol{R}\ge 0$ selected proper way, i.e. Bryson’s Rule, or principle of unit weighting, or finally, heuristic setting of the weighting matrices can be used.

In some cases, so as to simplify and minimize workload on the selection of the weighting matrices $\boldsymbol{Q}$ is set to the identity matrix $\boldsymbol{I}$, and $\boldsymbol{R}$ is set to zero. Regarding [4] the MPC performance dominantly depends of p (poles of the Laguerre functions) and N (number of orthonormal functions). In other words, selection of weights of $\boldsymbol{Q}$ and $\boldsymbol{R}$ is not necessary to archive.

It is well-known that cost function can be re-written as function of η instead of $y(T_i+T)$. With the assumption that UAV reference trajectory $r(t)$ would not change within the prediction horizon of $T+p$ the quadratic integral performance index (cost function) can be expressed as:

$$J=\boldsymbol{\eta}^T\prod\boldsymbol{\eta}-2\boldsymbol{\eta}^T\{\boldsymbol{\Psi}_1\boldsymbol{r}(T)-\boldsymbol{\Psi}_2\boldsymbol{X}(T)\}+\int_0^{T_p}\boldsymbol{w}^T(T_i+T)\boldsymbol{Q}\boldsymbol{w}(T_i+T)dT\rightarrow Min \tag{34}$$

In Eq(34):

$$\prod\quad=\int_0^{T_p}\boldsymbol{\phi}(T_i)\boldsymbol{Q}\boldsymbol{\phi}^T(T_i)dT+\overline{\boldsymbol{R}} \tag{35}$$

$$\boldsymbol{\Psi}_1=\int_0^{T_p}\boldsymbol{\phi}(T_i)\boldsymbol{Q}dT \tag{36}$$

$$\boldsymbol{\Psi}_2=\int_0^{T_p}\boldsymbol{\phi}(T_i)\boldsymbol{Q}\boldsymbol{C}\boldsymbol{e}^{\boldsymbol{A}T}dT \tag{37}$$

$$\overline{\mathbf{R}}=diag(\lambda_i\boldsymbol{I}_{N_i\times N_i}) \tag{38}$$

In Eq(38): λ_i are eigenvalues of the extended system matrix $\boldsymbol{A}$, and $\boldsymbol{I}_{N_i\times N_i}$ is an identity matrix of the dimensions of $N_i\times N_i$.

The minimum of the cost function (34) with no hard constraints on any variables can be determined using the least squares technique [4]:

$$\boldsymbol{\eta}=\prod^{-1}\{\boldsymbol{\Psi}_1\boldsymbol{r}(T)-\boldsymbol{\Psi}_2\boldsymbol{X}(T)\} \tag{39}$$

The derivative of the control input can be determined as:

$$\dot{\boldsymbol{u}}(T)=\begin{bmatrix}L_1^T(0) & 0 & \dots & 0\\ 0 & L_2^T(0) & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & L_r^T(0)\end{bmatrix}\prod^{-1}\{\boldsymbol{\Psi}_1\boldsymbol{r}(T)-\boldsymbol{\Psi}_2\boldsymbol{X}(T)\} \tag{40}$$

Integrating Eq(40) yields to:

$$\boldsymbol{u}(t)=\int_0^t\dot{\boldsymbol{u}}(T)dT \tag{41}$$

The continuous time MPC systems closed loop stability might be ensured by adding some weighting to the system terminal state in cost function of (35). The integral performance index (cost function) is a quadratic function, and hard constraints can be put easily to the system predicted output $\boldsymbol{x}(t)$, to the first derivative of the control input $\dot{\boldsymbol{u}}(T)$, and, finally to the control input $\boldsymbol{u}(t)$ required.

To form a set of inequality constraints needed to solve quadratic optimization problem of the cost function (35) requires discretization of the trajectories. Let us set bounds on derivative of the control input as follows:

$$\dot{\boldsymbol{u}}_{low}(T_i+T)\leq\begin{bmatrix}L_1^T(T_i) & 0 & \dots & 0\\ 0 & L_2^T(T_i) & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & L_r^T(T_i)\end{bmatrix}\eta\leq\dot{\boldsymbol{u}}_{high}(T_i+T) \tag{42}$$

Eq (42) denotes the set of linear inequality equations. In Eq (42) $T_i, i=1,2,3,\dots$ denotes future time instants at which limits on $\dot{\boldsymbol{u}}(T)$ are imposed. Since $\boldsymbol{L}_k(T); k=1,2,3,\dots,r$ are exponential functions guaranteeing exponential decay of $\dot{\boldsymbol{u}}(T_i+T)$, it is necessary to set constraints only on the initial stage of the prediction horizon *p*, which can reduce number of the constraints required.

Constraints set on the control signal, the system predicted output variables and system states can be determined as follows below [1, 2, 4]:

$$u_{low}(T_i+T) \leq \begin{bmatrix} \int_0^{T_i} L_1^T(\gamma)\,d\gamma & 0 & \dots & 0 \\ 0 & \int_0^{T_i} L_2^T(\gamma)\,d\gamma & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \int_0^{T_i} L_r^T(\gamma)\,d\gamma \end{bmatrix} \boldsymbol{\eta} + \qquad (43)$$

$$\boldsymbol{u}(T_i - T) \leq u_{high}(T_i+T)$$

where $\boldsymbol{u}(T_i - T)$ is the previous control signal. Regarding experiences gained from computer simulations with pre-chosen p and N we have:

$$\int_0^{T_i} L_k(\gamma)\,d\gamma = \left[\boldsymbol{A}_p^{-1}\left(e^{\boldsymbol{A}_p T_i} - \boldsymbol{I}\right)\boldsymbol{L}(0)\right]^T \qquad (44)$$

where $\boldsymbol{A}_p$ is defined by Eq (16). For a given set of time instants of T_i equation (43) yield to the set of linear inequality constraints on control input signal, say:

$$u_{low}(T_i+T) \leq e^{\boldsymbol{A}_p T_i} + [\boldsymbol{I}_{int}^1(T_i) \quad \boldsymbol{I}_{int}^2(T_i) \quad \dots \quad \boldsymbol{I}_{int}^r(T_i)]\eta \leq u_{high}(T_i+T) \qquad (45)$$

And so we have predicted output of:

$$x_{low}(T_i+T) \leq e^{\boldsymbol{A}_p T_i} + [\boldsymbol{I}_{int}^1(T_i) \quad \boldsymbol{I}_{int}^2(T_i) \quad \dots \quad \boldsymbol{I}_{int}^r(T_i)]\eta \leq x_{high}(T_i+T) \qquad (46)$$

The procedure described above assumed that all the states at any sampling time of T_i are known. Many cases, due to any reasons, it is impossible to measure all the states of the plant to be controlled, i.e. the state estimation is required to estimate state variables of $x(T_i)$. In this case, the continuous time MPC control system will have following estimator equation [1, 2, 4]:

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{B}\dot{\boldsymbol{u}}(t) + \boldsymbol{J_{obs}}[\boldsymbol{y}(t) - \boldsymbol{C}\hat{\boldsymbol{x}}(t)] \qquad (47)$$

In Eq (47) $\hat{\boldsymbol{x}}(t)$ is estimated value of $\boldsymbol{x}(t)$, and, $\boldsymbol{J_{obs}}$ is the observer gain matrix calculated recursively, and off-line, i.e. no direct need of solution of Ricatti equation. Derivative of the control input $\dot{\boldsymbol{u}}(t)$ can be determined from the optimal solution of the MPC strategy.

The observer can be designed using the static Kalman-filter standard technique. Supposing that the UAV spatial motion model (**A**,**B**,**C**,**D**) is completely controllable and observable, gain matrix of $\boldsymbol{J_{obs}}$ can be chosen such that the error of the estimation of $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$ will decay exponentially at any desired rate and at any desired time. Worth to mention that the observer static gain matrix of $\boldsymbol{J_{obs}}$ is often limited due to existence of such measurement noises [1, 2, 4, 11, 12].

7. CONTROL LAW SYNTHESIS FOR UAV SISO MPC SYSTEM

In general, small UAV dynamics is considered for rigid body, linear model expressed either in MIMO (state space model), or in SISO (transfer function) forms. For further discussion of MPC control of small UAVs, the aerodynamic model of the lateral motion of the fixed-winged Trainer-60 SUAV was used is as follows [9]:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} = \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -0,7724 & 0 & -18,9671 & 9,0867 \\ 1,9247 & -19,9149 & 7,7565 & 0 \\ 69,1314 & -23,8689 & -2,5966 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} 0 & 2,2582 \\ -23,8289 & 1,5015 \\ -11,7532 & -15,2855 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}. \tag{48}$$

In Eq (48): v is the lateral translational speed, p is the roll rate, r is the yaw rate, ϕ is the roll angle position, δ_a is the angular deflection of the ailerons, and, finally, δ_r is the change in rudder angular position.

In [9] the UAV lateral motion dynamic model was reduced to that of the short period motion dynamic model, i.e.:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} = \begin{bmatrix} \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -19,9149 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p \\ \phi \end{bmatrix} + \begin{bmatrix} -23,8289 \\ 0 \end{bmatrix} \delta_a \tag{49}$$

The UAV spatial motion represented by Eq (49) is often subjected to some plant disturbance. Leaning on these conditions, the control free UAV can be represented as follows:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} = \begin{bmatrix} \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -19,9149 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p \\ \phi \end{bmatrix} + \begin{bmatrix} -23,8289 \\ 0 \end{bmatrix} \delta_a + Y_d d \tag{50}$$

The control free UAV model given by Eq (50) can represented with its Laplacian model, and the block diagram was constructed and it is depicted in Fig. 3.

From Fig. 3. it is easily can be seen that the UAV lateral short period motion model is the SISO one. The plant disturbance filter transfer function is as follows below:

$$Y_d(s) = \frac{0,1}{0,01s+1} \tag{51}$$

From Fig. 3. the UAV roll rate dynamic model subjected to the plant disturbance (default) can be derived as it is given below:

$$p_n(s) = \frac{A}{1+sT}\delta_a(s) + Y_d(s)D(s) = \frac{1,1965}{0,0502s+1}\delta_a(s) + \frac{0,1}{0,01s+1_d}D(s) \tag{52}$$

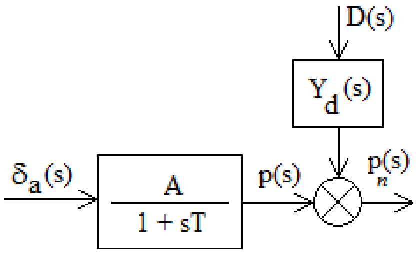


FIG. 3. Block diagram of the SISO UAV model.

The MPC system design problem can be formulated as follows: for the dynamic system illustrated in Fig. 3. design the controller ensuring closed loop control system dynamic performances as they derived by [10].

Design parameters of the MPC system has chosen using [2, 11, 12] as follows below:

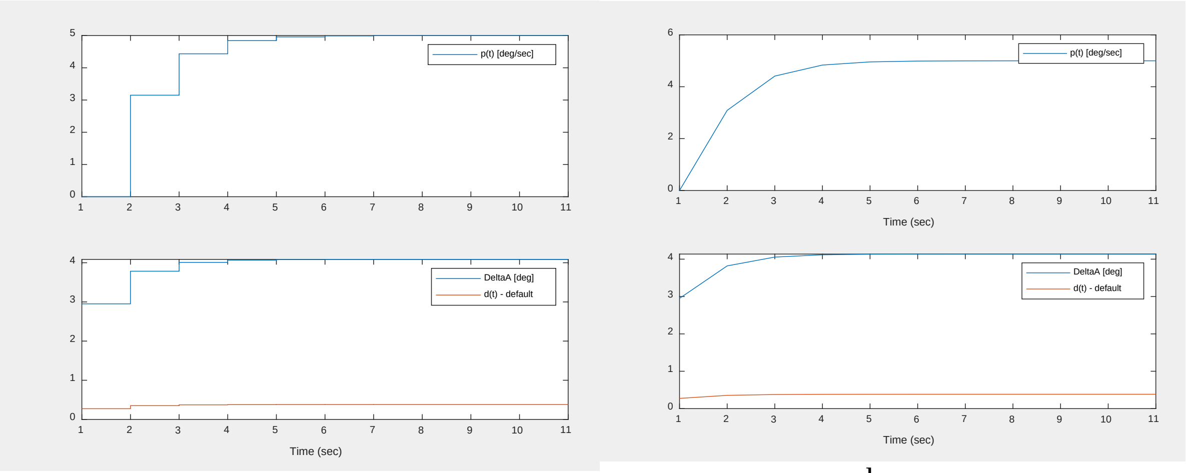
1) Roll rate reference: 5 deg/s;
2) Sampling period: $\Delta t = 0{,}1\ sec$;
3) Settling time:$t_s = 6\ sec$;
4) Model horizon N: $N\Delta t = t_s$; N=60;
5) Control horizon: M=5;
6) Prediction horizon: P=50;
7) ***Q***=1;
8) ***R***=[1 1].

Let us consider a UAV flight scenario of the collision avoidance, whilst UAV is forced to change directional angle suddenly to avoid hitting any object being either natural or artificial. For that mission, to have fast responses from the UAV, control of the roll angle is required, to maintain roll angle position required. This method is widely used in automatic flight control of both manned and unmanned aerial vehicles.

The reference model of the small UAV roll rate behavior to be followed by the closed loop control system has been chosen to be:

$$p_m(t) = 5 * 1(t)\ \text{deg/s} \tag{53}$$

Using data defined above, the UAV roll rate closed loop control system has been designed and tested in time domain using `cmpc.m` built-in function of the MATLAB Model Predictive Control Toolbox [14, 15, 16]. Results of the computer simulation are depicted in Fig. 4.a. using stairs plotting option, and in Fig. 5.b. using conventional plotting option offered by MATLAB®.



a. b.

FIG. 4. Small UAV rolling motion MPC - closed loop control system step response.

Fig. 4 demonstrates that the roll rate reference given by Eq (53) is followed, and for the settling time range of t_s it is reaching final value of the roll rate of 5 deg/s. For further scheduling and augmentation of the UAV closed loop control system dynamic performances one can change control horizon M, prediction horizon P, and weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ of the integral performance index given by Eq (33).

8. CONCLUSIONS AND FUTURE WORK

The motive behind this research work was to summarize mathematical backgrounds serving for solution of the MPC design problems. The approach of predictive thinking about UAV flight path planning can ensure that future control input required to minimize error between the set point (reference path to be followed) and predicted output in least square means will be an optimal solution to a standard cost function minimization problem. Leaning on this technique UAV flight path design and flight via path designed can be optimized. However, the parameter setting of M and P, and parameter selection of $\boldsymbol{Q}$ and $\boldsymbol{R}$, is requiring a complex set of dynamic performances of the UAV closed loop automatic flight control system, such as time domain performance indices (peak time, settling time, percent overshoot), and frequency domain performance indices (damping ratio, gain margin, phase margin), which are often unknown or, not defined ones, and validation of the results of the computer simulation requires some preliminaries serving to define set of performance indices of the UAV closed loop control system.

The problem introduced for the MPC system design will be extended firstly for more sophisticated reference input signals like exponentials of the flare flight phase of the UAV landing. Moreover, the test input signals of the UAV closed loop automatic flight control system will be chosen for typical flight phases of the UAV collision avoidance maneuvers in any relationships, like UAV vs UAV, or UAV vs non-UAV.

The competing UAV dynamic model available is the state space model to that of the applied transfer function models, and, for further preliminary computer aided design the multivariable model will be used to design closed loop MPC control systems.

Finally, the default external disturbance model provided for use by *`cmpc.m`* built-in function of MATLAB will be substituted with random turbulence models ensuring more realistic representation of the stochastic atmospheric turbulences.

REFERENCES

[1] Löfberg, J. (2001): *Linear Model Predictive Control – Stability and Robustness.* Linköping University, Division of Automatic Control;

[2] Seeborg, D. E. et al. (2005): *Process Dynamics and Control.* John Wiley & Sons, Inc., 2nd Edition;

[3] Békési Bertold, Szegedi Péter: *Preliminary Design of Controller of Longitudinal Motion of the Unmanned Aerial Vehicle Using LQR Design Method*. Proceedings of the 10th International Conference: Transport Means 2006, Kaunas, Lithuania, pp. 324-327;

[4] Raemaekers, A.J.M.: *Design of a model predictive controller to control UAVs*. DCT 2007.141, Technical University of Eindhoven, 2007;

[5] Békési Bertold, Wührl Tibor: *Redundancy for micro UAVs – control and energy system redundancy*. Proc. of the International Conference Deterioration, Dependability, Diagnostics 2012, Brno, Czech Republic, pp. 123-130. (ISBN:978-80-7231-886-5);

[6] V. Șandru, M. Rădulescu: The Use of UAV's During Actions of Integrated Air Defense Systems. *Review of the Air Force Academy*, No 3 (30) 2015, pp (133-138), 2015;

[7] S. Pop, A. Luchian, R. G. Zmădu, E. Olea: The Evolution of Unmanned Aerial Vehicles. *Review of the Air Force Academy*, No.3 (35)/2017, pp(125-132), 2017;

[8] http://engineering.utsa.edu/ataha/wp-content/uploads/sites/38/2017/07/EE5143_Module9.pdf (Accessed:28 February 2019);

[9] Prof. Dr. Róbert Szabolcsi: Optimal PID Controller Based Autopilot Design and System Modelling for Small Unmanned Aerial Vehicle. *Review of the Air Force Academy*, No.3 (38)/2018, pp(43-58);

[10] R. Szabolcsi, Lateral/Directional Flying Qualities Applied in UAV Airworthiness Certification Process. *Land Forces Academy Review*, 3/2014:(75) pp(336-346), 2014;

[11] Maciejowski, J.M: *Predictive Control with Constraints*. Prentice Hall, Upper Saddle River, NJ, 2002;

[12] Rawlings, J.B.: Tutorial Overview of the Model Predictive Control. *IEEE Control Systems Magazine*, 20(3), 38 2000;

[13] Dávid László, György Katalin, Kelemen András: Comparisons Between Applied Model Predictive Control, State Dependent Riccati Equation, and Finite Horizon Discrete Optimal Control Algorithms. *V. Műszaki Tudományos Ülésszak, Kolozsvár, 2014. Műszaki Tudományos Közlemények 2.*, pp(61-74). https://eda.eme.ro/xmlui/handle/10598/28549 (Accessed: 1 March 2019.);

[14] MATLAB® R2018b, User's Guide, The MathWorks, 2018;

[15] MATLAB® R2018b Control System Designer/Control System Toolbox 10.3, User's Guide, The MathWorks, 2018;

[16] MATLAB® R2018b Model Predictive Control Toolbox, User's Guide, The MathWorks, 2018;

[17] V. Prisacariu, M. Boşcoianu, A. Luchian: Innovative Solutions and UAS Limits. *Review of the Air Force Academy*, No 2 (26) 2014, pp(51-58), 2014;

[18] L. Gherman: An Electromagnetic Launch System for UAVs. *Review of the Air Force Academy*, No 2/2012, pp(5-11), 2012.