# MATHEMATICAL MODEL AND NUMERICAL SIMULATIONS FOR PHOTOVOLTAIC PANELS

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**Abstract:** This paper presents some problems concerning efficient using of photovoltaic cells in present energetic systems. An important aspect in now days is modeling and numerical simulation for systems with photovoltaic cells. For this reason one used a mathematical model in literature to implement a simulation scheme in MATLAB/SIMPOWERSYS for photovoltaic panels. One start from a simplified mathematical model for a photovoltaic cell and next one extends this model to a photovoltaic panel with mxn cells. Implemented model in MATLAB/SIMPOWERSYS is then used for numerical simulation of the panel with a fixed load and variable irradiation and for simulation of a hybrid system with photovoltaic panel and battery.

*Keywords:* photovoltaic panels, mathematical model, numerical simulation, numerical simulations

## **1. INTRODUCTION**

Politico-economic conditions in last few years at global level concerning energetic resources are characterized by two pressing problems which direct the entire world to replace other forms of energy with electric energy. By one hand greenhouse gases generated using fossil combustibles and by other hand the narrow fossil combustibles depletion leads to intense research in alternative energy sources domain at global level, especially in electric energy alternative sources. Use of wind power, tidal and wave energy and photovoltaic energy mark visible progresses in the last period.

Important progresses are obtained also in electric power storage. Li-Ion and Li-Polymer batteries with very high capacities permitted together with photovoltaic energy sources to complete a flight around the earth, using only electric energy [see Solar Impulse]. Such a performance would not be possible with technology 10 years ago.

Towards replacement of fossil combustibles with electric energy one observes increasing the weight of electrical energy used by household consumers provided by wind turbines or photovoltaic panels of relative small power, which can ensure the energy for housework. In the maximum production period of these small power plants they can uphold even national energetic systems of some countries which widely use such systems (Germany, Italy, Japan, USA).

A second big domain which is a huge consumer of fossil combustibles is transportation. One remarks also in this field apparition of hybrid cars with over 600 km autonomy or even complete electric cars with autonomy of hundreds km. The urban transportation aims to use electrical busses, implemented already in some cities like Turin or some university campuses like Oxford. Aeronautic transportation is by excellence a huge consumer of fossil combustibles. Here also appeared in last few years researches to replace fossil combustibles with electric energy. It is a long way to obtain an electric airliner but UAV of smaller or bigger dimensions, drones, last generation surveillance systems use more and more electric energy stored in last generation Li-Ion batteries. Airbus concern already built a light aircraft, one person on board, one hour autonomy, powered only by electric energy. It already passed the famous limit of English Channel overflight [1]-[5].

## 2. STRUCTURE OF AN ELECTRIC SYSTEM WITH PHOTOVOLTAIC PANELS

In order to use the photovoltaic energy, the basic element which generates this energy - the photovoltaic cell – has to be placed in a system capable to use this energy. Such a system is presented in fig.1.



FIG. 1. Component elements of a system with photovoltaic panels

### Photovoltaic cell

Basic elements of photovoltaic panels are the solar (photovoltaic) cells. Majority of solar cells are manufactured of silicon, but also other materials are used. Solar cell is based on photoelectric effect that means the ability of some semiconductors to transform electromagnetic radiations in electric current.

A solar cell is in fact a p-n junction with two different silicon layers with two types of impurities. N layer is unpurified with atoms with more than one valence electrons – donor atoms, and p layer contains less valence electrons – acceptor atoms. When these two layers are put together, near the separation surface, valence electrons from n layers spread in p layer, leaving behind, a positive charged zone of donors. Holes from p layer spread in n layer, leaving behind, a negative charged of acceptors. By this way an electric field appears between these two layers, representing a potential barrier for the supplementary fields. When the electrons and holes can't anymore pass through this barrier, a balance appears. Electric field pushes the electrons and holes in opposite directions, so that the electric current can flow only in one direction. Electrons can move from p layer to n layer and holes in inverse direction.

Both ends are joined metallic contacts to collect electrons and holes so the electric current can flow. N layer is exposed to the solar radiation. Contacts are manufactured of many metallic strips, permitting light to reach the solar cell. These are known as fingers.

Functioning principle of the photovoltaic cell can be described as follows: solar radiation photons blow the solar cell. Three situations can appear: non-reflected photons reach the substrate. Those with less energy pass through solar cell with no effect. Only photons with sufficient energy, over the band zone of silicon, can produce an electronhole pair. These pairs are generated in both halves of p-n junction.

Electrons from p layer and holes from n layer pass through the junction and are transmitted in opposite directions by the electric field. This is the light generated current. This current depends on the irradiation: if the irradiation is intense, contains many photons with enough energy to generate electron-holes pairs, so the solar cell generates more electric current.

For solar cells manufacturing in most cases silicon is used. Although other techniques are developed, silicon is preferred in more than 80% of situations. It is a widespread material in the earth, can be found as silicon dioxide and is not toxic.

In literature, two silicon solar cells types are known: mono-crystalline and polycrystalline. There is a third type – amorphous silicon, but their efficiency in less than first two types.

Other solar cells are manufactured of Cooper – Indium – Gallium – Selenium (CIGS) or Cadmium Telluride (CdTe). A great research effort is involved to find new photovoltaic materials, but at this time, there are no commercial substitutes for the solar cells presented above.



FIG.2. Simplified model of a solar cell

Electric equivalent scheme of a solar cell is presented in fig. 2. Electric current through the solar cell I described by the equation [6]-[10]

$$I = I_{ph} - I_s \left( e^{\frac{V}{mV_t}} - 1 \right), \tag{1}$$

where *I* – current through the solar cell; *Is* – inverse saturation current of the diode; *Iph* – photovoltaic generated current; *V* – voltage applied to the diode;  $V_t$  – thermal voltage; *m* – diode ideal factor,  $m = 1 \div 5$ .

Thermal voltage is determined by the relation

$$V_t = \frac{K \cdot T}{e},\tag{2}$$

where K – Boltzmann constant  $1.38 \cdot 10^{-23}$  J/K , T – absolute temperature [K]; and e-charge of the electron.

A PV panel is composed from many solar cells, connected in series and/or in parallel in order to obtain output voltages and currents compatible with the electric load.

Irradiation and temperature are two important parameters affecting the photovoltaic panel characteristics. As it was mentioned before, photovoltaic current is proportional with the irradiation level, so an irradiation increase produce a bigger photovoltaic current.

More, the short-circuit current is proportional with the photovoltaic current, so the short-circuit current is proportional with the irradiation level. By other hand, the effect of the irradiation upon the open-circuit voltage is small, as the dependence of the light generated current is logarithmic, like in equation (1).

In practice, voltage dependence of irradiation is negligible. Because the irradiation effect upon the current and voltage is positive, both raise, so the irradiation effect upon the generated power is also positive – more irradiation, more power.

By other hand, temperature influences the voltage. Open circuit voltage depends linear by temperature. Photovoltaic cells producers presents in catalogue papers, specific technical parameters: open circuit voltage, short-circuit current and maximum power, when temperature varies.

One has to mention that temperature and irradiation depend on atmospheric conditions that are not constant in a day and in a year.

Photovoltaic cells produced in present have a very small power, around 4 W maximum and the voltage at the photovoltaic cell terminals is about the opening voltage of a diode, that means 0.6 V.

In order to obtain enough energy for an application it is necessary to use many solar cells linked in series or in parallel. By this way, the superior element appears – the photovoltaic panel. Depending the connection between solar cells one obtain a certain open circuit voltage and a certain short-circuit current. Voltage-current characteristic of the photovoltaic panel will reproduce on the scale the characteristic of a solar cell, depending the number of solar cells disposed in series or in parallel. In order to obtain a higher voltage one has to connect cells in series till one obtains the desired voltage. In order to obtain a higher current one connects the solar cells in parallel till obtains the desired current. Theoretically, connecting solar cells one can obtain any output current and voltage, but one has to take in account that in fact any solar cell is a diode. If one of them is interrupted, it will shut down all the cells connected in series with it. So it is preferable to manufacture photovoltaic panels for smaller voltages and higher currents following to obtain higher voltages through power converters.

Characteristics of one solar cell is presented in fig. 3.



FIG.3. Characteristics of a solar cell: a. - voltage-current dependence; b. - power-voltage dependence

In fig. 3 one can observe a very disadvantageous behavior of the solar cell and by consequence of the photovoltaic panel. Beginning from a certain consumer power, the current drops very fast and also the output power decrease to zero. In order to be used efficiently in a system, photovoltaic panels, which reproduce in scale the characteristics of a cell, are provided with an electronic block named Maximum Power Point Tracker (MPPT).

This block maintains an impedance to the input bigger or equal with that produce the maximum power of the photovoltaic panel. By this way one prevents the current drop to the input and implicit the premature decrease of the output power. If the external load increase in power, MPPT will provide to the output only the maximum power the panel can produce at that irradiation. MPPT block can contain also a power converter to bring the panel output voltage to the consumers input voltage.

Control algorithms used in MPPT construction are many types in present. The most used algorithms are hill-climbing type which follow to modify the functioning point of the panels in the increasing power direction till the maximum output power is achieved. Other algorithm are based on fuzzy logic or neural networks.

### 3. SIMPOWERSYS IMPLEMENTATION OF A SOLAR CELL MODEL

In view to implement the mathematical model of a photovoltaic cell in SIMPOWERSYS one started from equation (1), where  $I_{ph}$  is equal to the short-circuit output current of the photovoltaic cell.

Relation (1) can be write in the form:

$$V = m \cdot V_t \ln\left(\frac{I_{ph} - I + I_s}{I_s}\right),\tag{3}$$

Knowing  $I_{ph}$  from cataloque data and the open circuit voltage of the solar cell, at the standard irradiation of 1000 W/m<sup>2</sup>, in equation (3) one can impose I=0 for the open circuit situation and one obtain

$$V_{0C} = m \cdot V_t \cdot \ln\left(\frac{I_{ph} + I_s}{I_s}\right),\tag{4}$$

And further

$$m = \frac{V_{oc}}{V_t \cdot \ln\left(\frac{I_{ph} + Is}{Is}\right)},\tag{5}$$

Based on relations (1) and (5) one built the model in SIMPOPWERSYS presented in figure 4, where the solar cell has a variable resistive load.



FIG.4. SIMPOWRSYS model of a solar cell

In order to develop the solar cell model, one could use a commanded current source unlike the commanded voltage source used in this model, but in this case SIMPOWERSYS does not accept to connect in series the models of a solar cells, to obtain a photovoltaic panel model. In consequence, it is more advantageous the model with voltage source from the SIMPOWERSYS using point of view and to connect with other devices like DC/DC power converters.

A photovoltaic panel which contains  $n_p$  blocks in parallel composed by  $n_s$  cells in series, can be equate with a solar cell with a certain open circuit voltage and short-circuit current.

As numerical example, one used first for a solar cell simulation the parameters of the SUNPPOWER C60 solar cell which has the open circuit voltage 0.682 V and short-circuit current 6.24 A at the standard irradiation and temperature  $25^{\circ}$ C. From relation (5) results m=2.11. One obtained voltage-current characteristic in figure 5, which is a good concordance with the catalog characteristic of this solar cell.



**FIG.5.** Obtained characteristics for SUNPOWER C60 solar cell; a. – voltage-current characteristic; b. – power-voltage characteristic

After that one followed to develop a photovoltaic panel with the open circuit voltage 24 V and the short-circuit current 31.2 A. For this panel results  $n_s=36$  and  $n_p=5$ . So, the photovoltaic panel should contain 36x5=180 solar cells. In view to simulate the panel functioning in SIMPOWERYS is totally inadequate to connect 180 cells like that in fig.4.

More appropriate is to use one single solar cell with equivalent parameters of the panel, so there will be  $I_{ph}$ =31.2 A and open circuit voltage 24 V. From equation (5) results m=67.99. Using the circuit with variable load in figure 4 one draw characteristics of the equivalent solar cell in fig. 6.



FIG.6. Photovoltaic panel characteristics: a. – Voltage-current characteristic; b. Power-voltage characteristic

For testing this model one built a complete system in figure 7. In this figure it is a solar cell (in fact an equivalent solar cell for the photovoltaic panel as one considered above), with a variable irradiation, followed by a constant load of 10  $\Omega$  (fig.7). Irradiation was composed by a medium value of 700 W/m<sup>2</sup> and the absolute value of a sinusoidal variation was superimposed to this medium value. The sinusoid was considered with the amplitude of 300 W/m<sup>2</sup> and pulsation 0.01 rad/s. In these conditions one obtained the load current variation in fig. 8.



FIG.7. Photovoltaic panel model with variable irradiation and constant load



FIG.8. Output voltage and current of the photovoltaic panel with variable irradiation

To test further the photovoltaic panel model one conceived a hybrid system composed from a photovoltaic panel, a double voltage DC/DC boost converter, and a battery in parallel with the converter output. Photovoltaic panel is the same used in the previous simulations and the battery was considered Li-Ion type with nominal voltage 43 V, capacity 10 Ah, initial state of charge 100 % with voltage 50 V, internal resistance 0.043  $\Omega$  (figure 9). One considered a relative small value for the battery capacity in view to surprise in a relative short time of simulation the commutation between battery and photovoltaic panel. One considered also a 5  $\Omega$  resistive load. Irradiation variation was considered the same as in figure 7. Simulation results are presented in figure 10.



FIG.9. Hybrid system composed form a photovoltaic panel, converter, battery and load



FIG.10. Currents and voltages in photovoltaic panel - battery - load system

One observes that till 3200 s load receives power from battery, photovoltaic panel current is zero. After 3200 s battery current decreases enough, the load receive power also from photovoltaic panel. After 5000 s functioning stabilizes, in the maximum irradiation periods the photovoltaic panel feeds the load and a part of its energy charges the battery. In minimum irradiation periods panel energy decrease and the battery push current in system sustaining the load feeding. After 5000s one stabilizes the current medium value through load. In the period 0 - 3200 s load current decreases constantly.

Concerning the voltages, the battery and the load are connected in parallel, so the battery and load voltages are equal. Up to 3200 s battery voltage decreases constantly and the photovoltaic panel voltage is not enough to push energy on the load.

In the period 3200 - 5000 s is a transition period, when due to battery voltage decrease, photovoltaic panel begins to push energy in the system. Now, the panel voltage presents a slightly decrease and after 5000 s the functioning regime stabilizes and the medium value of the panel voltage rests constant and upon it is superposed the pulsations due to the variable irradiation.

One has to remark that in figures 10 a. and 10 b. appears panel voltages and currents, before the converter.

One tested the possibility to connect directly the panel in parallel with the battery, but due to the sharp decrease of the photovoltaic panel voltage with respect the current (see figure 6.a) the system presents fast oscillations and simulation speed drops drastic. In this condition the model is not useful anymore for long time simulations like in figure 10. Converter input characteristic avoids the panel output voltage to drop and the simulations can be performed with increased speed and the model is useful for long time simulations. Converter output characteristic is compatible with that of the battery so the transition from positive to negative current through battery, in dependence with the energy produced by the panel, is made without oscillations and simulation speed is very high.

This aspect is in concordance with practice, because on the photovoltaic panel output one uses a MPPT block which keeps the panel in the maximum power point, even if the load requested power increases and could drive the panels in the region of fast voltage dropping. By this way, panel pushes at one moment in the system only the maximum power produced by the irradiation in that moment, without a total voltage drop to output. One can say the boost converter behaves as a MPPT, even it is not a MPPT.

#### CONCLUSIONS

Using the literature formulae which describe the simplified functioning of a photovoltaic cell one obtained a simulation model in MATLAB/SIMPOWERSYS which behaves very good, describing exact enough the functioning of a solar cell.

Aiming to simplify the simulation of photovoltaic panel systems which contains many solar cells, one replaced a panel with  $n_s$  cells in series and  $n_p$  cells in parallel with one single solar cell with the short-circuit current  $n_p$  times bigger than that of one cell and the open circuit voltage  $n_s$  times bigger than that of one cell. One drawn the panel characteristics in this hypothesis and one performed simulations in SIMPOWERSYS. By simulations one tested the model to a variable irradiation, obtaining realistic results. One can say the obtained model is useful in hybrid systems simulation with photovoltaic panels, fuel cell and batteries.

Aiming photovoltaic panel use in hybrid system with a battery, one need to use a DC/DC boost converter to play MPPT role and to stabilize system functioning and keep simulation at high speed, compatible with long time simulations.

As future researches, in order to complete the instrument set for numerical simulations of photovoltaic panel systems it is necessary also the development and implementation of a mathematical model for a MPPT.

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