WING LIFT-DRAG RATIO OPTIMIZATION

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DOI: 10.19062/2247-3173.2018.20.24

Abstract: *Many important performances are obtained in flight at maximum aerodynamic finesse, such as maximum endurance and maximum climb angle for jet-powered airplanes, maximum range of propeller-driven airplanes, maximum power-off glide ration (for both jet-powered airplanes or for propeller-driven airplanes. The Prandtl lifting-line theory (LLT) was used to calculate lift and drag for a wing, and optimum combination of twist and incidence angle was found to maximize the aerodynamic finesse.*

Keywords: *lifting-line theory, lift, drag, twist angle*

1. INTRODUCTION

Any airplane design starts with selection of the principal components. In this preliminary design, the geometrical dimensions are analyzed this the aerodynamics characteristics to find the suitable combination.

Airfoils shapes are designed to provide high lift values at low drag for given flight conditions. More parameters, such aspect ratio or taper ratio, influence the overall lift (L) and drag (D) of the wing. The Reynolds number is also very important for airfoil performance. This number determines the achievable section maximum lift coefficient and lift-to-drag ratio. It also bounds the maximum thickness-to-chord ratio, beyond which point the airfoil will have unacceptable performance. Airfoils provide two-dimensional lift, drag and pitch momentum, which is equivalent to the characteristics of a section of an infinite span wing. Real wings, the wing with finite span, behave quite differently.

Several theories were developed to estimate the wing aerodynamic proprieties and their distribution, like Prandtl lifting-line theory, vortex lattice method, Trefftz plane analysis, the panel methods or CFD.

The Prandtl lifting-line theory (LLT) predicts the lift distribution through a Fourier sine series. The idea of this method, also known as Lanchester–Prandtl wing theory [1], is that the vortex loses strength along the whole wingspan because it is shed as a vortex-sheet from the trailing edge, rather than just at the wing-tips. The advantage of this method is easy implementation, low computational effort and, the most important, satisfactory accuracy for numerous problems. The disadvantage is consists in single wing calculation, with some restrictions imposed to the wing geometry.

The vortex lattice method (VLM) permit the evaluation of more complex configurations than the LLT because include also wing sweep and dihedral angle into the model. Further details can be captured, like fuselage lift and side-force contributions or he downwash of the wing on the tail.

The Trefftz plane analysis theory evaluates the lift distribution in the Trefftz plane, the plane perpendicular to the direction of flight assumed to be at an infinite distance behind the airplane [2,3]. The advantages of this analysis method are the automatically determination of the optimal trimmed lift distribution for minimum induced drag at a given CL. The Trefftz plane analysis does not require knowledge of the elevator deflections, contrary to LLT and VLM who have to calculate the appropriate elevator deflections to ensure trim. But this theory does not determine how this optimal lift distribution is attained (if ever), it miss or give incomplete information for the angle of attack, twist and camber influence on the optimal lift distribution.

The panel methods splits the entire geometry of the airplane into rectangular panels. Typically, the fuselage outer mold line is divided into numerous panels around the perimeter and multiple segments along the length [4]. Also, the surfaces of wing and tail are paneled individually, rather than simply modeling the camber line with the VLM method.

The computational fluid dynamics method (CFD) suppose the calculus of the domain surrounding the aircraft. This domain is divided into volume elements, often with variable density, the greatest being where the greatest flow condition variation is expected. The method calculates the flow properties at every volume cell using inviscid Euler equations or viscous Navier–Stokes equations. The Euler method has a much faster run time, but is not capable of predicting drag directly. The absolute drag values produced by CFD must be carefully analyzed because the most codes do not predict transition from laminar to turbulent flow, and so the transition location must be input directly. Without a reliable method of determining the transition, the drag predictions are often unreliable.

2. THEORETICAL ASPECTS

If the infinite aspect ratio (two-dimensional) lift–incidence relation is linear [5]

$$C_{L,\infty} = f(\alpha_\infty) \tag{1}$$

The vortex structures trailing downstream of a finite wing produce an induced downwash field near the wing which can be characterized, according LLT theory, by an induced angle of attack (FIG. 1). For a finite aspect ratio, *AR*, with elliptic loading the induced incidence is

$$\alpha_i = \frac{C_L}{\pi AR} \tag{2}$$

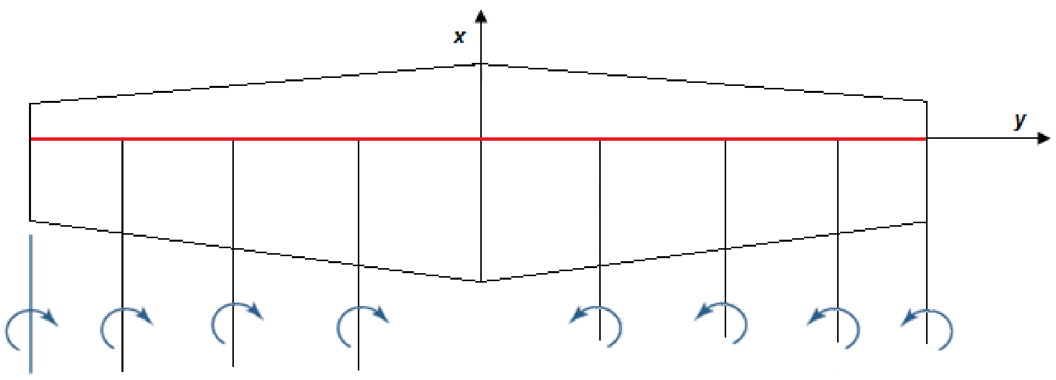


FIG. 1. Lifting-line horseshoe vortex representation

and at geometric incidence α the lift coefficient is that for the infinite aspect ratio at geometric incidence

$$\alpha_\infty = \alpha - \alpha_i \tag{3}$$

So, if the wing is untwisted, the downwash and induced incidence are uniform along the span for the elliptic loading.

Taking account the induced incidence, the lift coefficient can be expressed by

$$C_L = \frac{a}{1 + \frac{a}{\pi AR}}(\alpha - \alpha_0) \tag{4}$$

where a is the two-dimensional lift–incidence slope.

An elliptically loaded wing's induced drag coefficient is

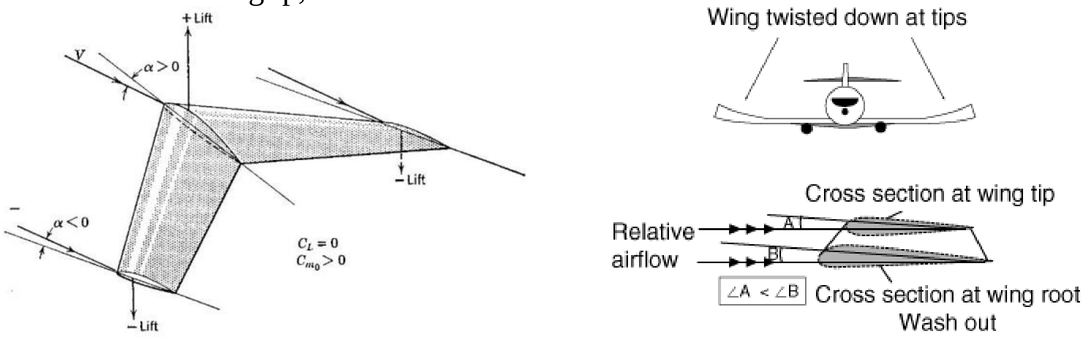
$$C_D = \frac{C_L^2}{\pi AR} \tag{5}$$

Practical wings are rarely constructed with an elliptic variation of chord length, since this is more expensive to manufacture than rectangular or trapezoidal planforms. Therefore. a corrected formula for untwisted unswept wings is used

$$C_D = \frac{C_L^2}{\pi e AR} \tag{6}$$

where e is an induced-drag factor which depends on the taper ratio and aspect ratio. The values of e are calculated (by LLT method, for example) or charted [6]. Anyway, the correction factor doesn't change the induced drag coefficient by more than about a tenth over the practical range of taper ratio and aspect ratio [7].

Most of real wings, mandatory for flying wings (FIG. 2a), have a twist angle over the span. That conduce to a lift redistribution to ensure the wing tip is the last part of the wing surface to stall. It means twisting the wingtip with a small amount downwards in relation to the rest of the wing. This ensures that the effective angle of attack is always lower at the wingtip than at the symmetry plane, so the root part will always stall before the tip part (FIG. 2b). This is because the aircraft's flight control surfaces, ailerons and flaps, are positioned at the wingtip, and we need those control surfaces to remain effective.



(a) (b)

FIG. 2. Wing twist concept (a) on a flying wing [8] (b), on an airplane [9]

3. MATHEMATICAL MODEL

The Prandtl lifting-line theory was selected as method for aerodynamic three dimensional wing identification [4,10]. Several reasons founded that selection. As already mentioned, speed of calculation and accuracy of results are some of method advantages. Also, the method is satisfactory for most aircraft configuration.

For example, despite the high degree of detail provided by the panel methods relative to LLT, in many practical cases, the number of panels required to analyze the aircraft is small and the advantage of the method vanish. More, since the target of this study is the wing alone, the LLT is the most appropriate for being used.

As derivation of the numerical lifting-line theory suitable for nonlinear lift-curve slopes [11], which is useful for analyzing wings near and beyond stall angle of attack, the present code is based on a matrix form was developed for subsonic applications [12].

The wing must meet the following criteria:

- The wing must have a negligible sweep (less than 10 degrees).
- The wing must have no dihedral.
- The wing must have at least moderate aspect ratio (more than 5).
- The flow is incompressible.
- The airfoils have linear lift-curve slopes and are not stalled.

The lifting-line method presented allows varied chord, camber, twist distributions along the span. The general formulation is

$$\frac{\pi c(\theta)}{2b}[\alpha + \alpha_{twist}(\theta) - \alpha_{0L}(\theta)]\sin(\theta) = \sum_{n=1}^{\infty} A_n \sin(n\theta)\left[\frac{\pi c(\theta) n}{2b} + \sin(\theta)\right] \tag{7}$$

where
A_n = influence coefficient
b_w = wing span
c = chord
α = angle of attack
α_{twist} = washout angle
α_{0L} = zero-lift angle of attack of the airfoil

The angle θ is a parameter for the semispan ratio at a distance y from the wing root.

$$\theta = \cos^{-1}\left(-\frac{2y}{b}\right) \tag{8}$$

The twisted wing is split into N segments between 0 and $\pi/2$ radians, and n assumes odd integer values from 1 to $2N - 1$. The chord at the given θ, $c(\theta)$, is can be found through linear interpolation across the semispan.

The LLT conduct to the following matrix equation to be solved for **x**:

$$\mathbf{Ax} = \mathbf{b} \tag{9}$$

Where

$$\mathbf{A}(i,j) = \sin[n(j)\theta(i)]\left\{\frac{\pi c(i) n(j)}{2b} + \sin[\theta(i)]\right\} \tag{10}$$

$$\mathbf{b}(i) = \frac{\pi c(i)}{2b}[\alpha + \alpha_{twist}(i) - \alpha_{0L}(i)]\sin[\theta(i)] \tag{11}$$

The parameter $n(j) = 2j - 1$ and the indices i and j go from 1 to N.

Solving equation (5), the section lift coefficient at station i can be calculate by

$$C_L(i) = \frac{4b}{c(i)}\sum_{j=1}^{N} x(j)\sin[\theta(i)] \tag{12}$$

Finally, the total lift coefficient of the wing and the induced drag are:

$$C_L = \pi AR x(1) \tag{13}$$

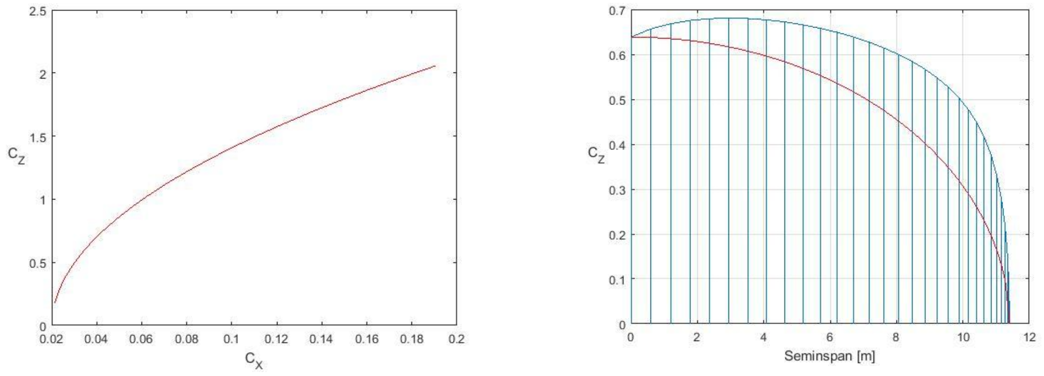
$$C_{Di} = \frac{C_L^2}{\pi e AR} \tag{14}$$

where $e = 1/\{1 + \sum_{j=2}^{N} n(j)[x(j)/x(1)]^2\}$ (15)

4. NUMERICAL RESULTS

A trapezoidal wing, with no sweep and dihedral angles and with constant twist over the span, was chosen for the analysis.

The LLT method was implemented into a MATLAB code in order to identify lift and drag for an aerodynamic surface. The first step was check the pertinence of results. A lift-drag diagram (FIG. 2a) was generated by varying the incidence angle. Also, the code permits to identify the lift variation over the span for -2 degree twist angle (FIG. 3b). Both variation are similar to theoretical results.



(a) (b)

FIG. 3. Code validation (a) Lift-Drag distribution, (b), Lift distribution over the wing

The geometric characteristics of the wing are taper ratio 0.25, aspect ratio 8 and surface aria of 65 m^2.

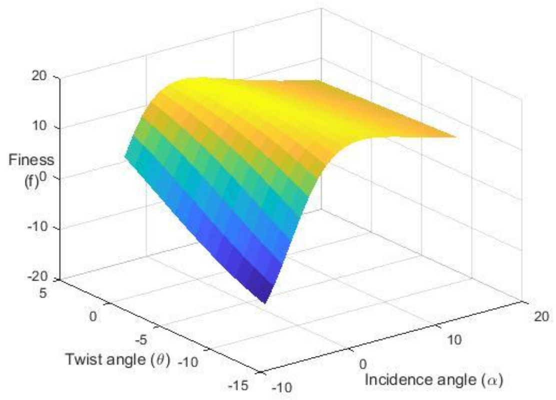
The airfoil aerodynamic characteristic of interest, the lift slope, is 6.9.

Since the aerodynamic finesse is the ratio between lift and drag, it is calculated as ratio between the coefficients found with LLT method ($f = C_L/C_D$). The Oswald coefficient calculated is also used into drag formulation.

Two parameters are chosen as variables: incidence angle and the twist angle. Should note that the constraint imposed to the twist angle is larger than the normal (in practice it is rarely over 5 degrees). The finesse is depending on those two parameters (FIG. 4).

An optimization problem is created in Matlab to find the maximum finesse. Since the optimization problems are minimum problem, the minimum of –*f* is searched.

Maximum value of the objective function is 17.67 and it is achieved at 6.23 incidence angle and 2.22 twist angle.



(a) (b)

FIG. 4. Lift-drag ratio (finesse) dependence on incidence angle and twist angle variation

5. CONCLUSIONS

One of the common constructive problems, the wing geometrical twist, is analyzed. It is difficult to vary the twist angle along the span, the practical solution is to use a constant angle. An optimum value is search in order to maximize the lift-drag ratio.

Matlab codes are use calculate lift and drag coefficients, to generate graphs and to create an optimization problem. Several methods (like Prandtl lifting-line theory, vortex lattice method, Trefftz plane analysis, the panel methods) was analyzed and LLT method was selected to be implemented.

The paper presents an easy and fast method of first level conceptual design. The method is useful to identify the basic parameters of an aerodynamic surface. Further detailed analysis should be made in order to obtain more refined results.

AKNOWLEDGMENT

This work was supported by a grant of the Romanian Space Agency, project nr.170/2017, STRATOTEST/ STAR Program for the 2012-2019.

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