AN $H_\infty$ DESIGN METHOD FOR THE PITCH ATTITUDE HOLD Autopilot OF A FLYING UAV

Adrian-Mihail STOICA, Petrisor-Valentin PARVU, Costin ENE

“Politehnica” University of Bucharest, Romania (adrian.stoica@upb.ro, parvupv@gmail.com, ene.costin27@gmail.com)

DOI: 10.19062/2247-3173.2016.18.1.24

Abstract: The paper presents an $H_\infty$ loop shaping method for the design of the automatic flight control system of an unmanned air vehicle’s (UAV’s) longitudinal dynamics. The design objectives include robust stabilization with respect to modeling uncertainties, pitch angle tracking of an ideal model and reduced sensitivity with respect to low frequency measurement errors. The design is illustrate by a case study for a flying wing UAV.

Keywords: UAV, pitch attitude hold autopilot, loop shaping, robust stability, ideal model tracking

1. INTRODUCTION

The applications based on Unmanned Air Vehicle’s (UAV’s) received much attention over the last decades. Among them, the flying wing configuration of the UAVs was intensively analyzed due to their advantages concerning the reduced drag and the fuel consumption. A disadvantage of this configuration with respect to the classical one is its instability which fact requires an automatic control system able to stabilize the aircraft and to provide acceptable maneuverability qualities. The design problem of such a system has been previously considered in the control literature (see e.g. [1], [2], [3]). The proposed design methods include both classical techniques based for instance on the well-known PID (Proportional Integral Derivative) control laws, optimal approaches based on the linear quadratic problem, the $H_\infty$ norm minimization and nonlinear methods including nonlinear inversion, etc ([4], [5], [6], [7], [8]). The methods mentioned above have advantages and drawbacks; the main difficulty in choosing the control design method is to ensure a trade-off between the complexity of the automatic flight control system and its performances. The actual applications require a wide spectrum of performances including not only the stabilization of the aircraft but also robustness with respect to modeling uncertainties and flying conditions changes, time response performances and reduced sensitivity to disturbances and measurement errors.

The aim of this paper is to present a design methodology for the Pitch Attitude Hold (PAH) autopilot of a flying wing configuration. The design procedure is illustrated and investigated for the Hirrus UAV designed and manufactured by a private Romanian company in collaboration with academics from Faculty of Aerospace Engineering of University “Politehnica” of Bucharest. The design approach is based on a modified version of the so-called two degrees of freedom (2 DOF) $H_\infty$ loop-shaping able to accomplish simultaneously several objectives as robustness stability, model tracking and disturbances attenuation requirements. The original 2 DOF $H_\infty$ method presented in [9] have been previously used for the autopilot design of the aircraft longitudinal and lateral
dynamics (see [10], [11], [12]). In the present paper the 2 DOF $H_\infty$ loop-shaping procedure is used for the design of the PAH autopilot of UAV with flying wing configuration. The paper is organized as follows: in the second section, the design objectives and the design model of the UAV are presented. The design procedure of the PAH autopilot is presented in the third section. The numerical results obtained in a case study for the Hirrus UAV are presented and analyzed in the fourth section. The paper ends with some final remarks.

2. DESIGN MODELS AND AUTOPILOT SYNTHESIS OBJECTIVES

The Hirrus platform illustrated in Fig. 1 has a flying wing configuration with the wingspan 3.2 m, length 1.2 m, maximum takeoff weight 6.5 kg, cruising speed of 80 km/h and with a maximum payload of 1 kg.

![Hirrus UAV](image)

The longitudinal dynamics of the UAV is approximated by the following linearized nominal model:

$$\dot{x}(t) = Ax(t) + B\delta_e(t)$$

where the state vector $x(t)$ includes the perturbational values of the longitudinal speed $u$, of the angle of attack $\alpha$, of the pitch rate $q$ and of the pitch angle $\theta$, respectively, with respect to their trimming conditions. The control input $\delta_e$ is the elevator deflection. The matrices arising in the above model corresponding to the nominal flight conditions $V = 15$ m/sec and $h = 10$ m are

$$A = \begin{bmatrix} -1.3374e-02 & 1.3589e+00 & 0 & -9.07036e+00 \\ 1.2592e-01 & -1.1151e+01 & -5.0437e+01 & -1.4134e+00 \\ -7.2028e+00 & 1.3765e+03 & -3.5594e+01 & 2.6574e-01 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -6.4712e-03 \\ 3.6575e-02 \\ -7.7492e+00 \\ 0 \end{bmatrix}$$

The eigenvalues of the state matrix from the above system are \{-2.3362e+01 \pm 2.6320e+02i; -1.6714e-02 \pm 1.1328e-01i\} showing that the UAV's phugoid poles are very close from the imaginary axis of the complex plane. This fact shows that in the absence of an automatic flight control system the time responses of the pitch angle are very slow. The measured outputs considered in this application are $y = [\alpha, q, \theta]^T$. The design objectives for the PAH autopilot are the following...
(DO1) Robust stabilization of the longitudinal dynamics;
(DO2) Zero steady state value for the tracking error $\theta_{com} - \theta(t)$ for piecewise constant values of the commanded pitch angle $\theta_{com}$;
(DO3) Shorter time responses of the pitch angle at constant commands;
(DO4) Reduced sensitivity with respect to low frequency measurement errors and robust stability with respect to modeling uncertainties.

3. THE DESIGN APPROACH OF THE PAH AUTOPILOT

In order to accomplish the design objective (DO2) defined in the previous section one introduces a reference model having the transfer function

$$H_m(s) = \frac{\omega_m^2}{s^2 + 2\xi_m\omega_m s + \omega_m^2}$$

where the natural frequency $\omega_m$ and the damping factor $\xi_m$ are chosen such that the settling time is much shorter than in the case of uncontrolled phugoid. Thus, for $H_m(s) = 10 / (s^2 + 0.8s + 10)$ the settling time is $t_s = 5.33$ sec and the damping ratio is $\xi_m = 0.12$. The control configuration used for the PAH autopilot is presented in Fig. 2.

![PAH Autopilot configuration with 2 DOF](image)

The two components $K_1$ and $K_2$ of the PAH autopilot are designed in order to accomplish the design objectives (DO1), (DO4). In the above figure, $W_1$ is a stable dynamic weighting and $W_2 > 0$ is a positive weighting, chosen in order to provide the sensitivity reduction to measurements errors and robustness stability performances. The dynamic weighting $W_1(s)$ as a low-pass filter aiming to improve the tracking performance at low frequency commands for $\theta_{com}$ (see e.g. [15]). The control system $K = [K_1 \ K_2]$ are determined such that it stabilizes the configuration from Fig. 2 and minimizes the $H_\infty$ norm of the mapping $u_1 \rightarrow y_1$, where

$$u_1 = \begin{bmatrix} \theta_{com} \\
W_1 \\
W_2 \end{bmatrix} \quad \text{and} \quad y_1 = \begin{bmatrix} z_1 \\
z_2 \\
z_3 \end{bmatrix}.$$
Reducing the $H_{\infty}$ norm of the dependence from $\theta_{\text{com}}$ to $z_3$ implies to accomplish the tracking performances of the reference model output. In fact the control configuration from Fig. 2 may be regarded as the following $H_{\infty}$-norm minimization problem.

![Diagram](image)

**Fig. 3.** The $H_{\infty}$-norm minimization problem

where $T$ denotes the so-called *generalized system* obtained from the Fig.2 taking into account that $u_2$ and $y_2$ coincide with the control input and with the measurement vector, respectively, namely $u_2 = \delta_e$ and $y_2 = [\alpha, q, \theta]^T$. The generalized system $T$ has the state-space realization of the following form

$$
\dot{x} = Ax + B_1u_1 + B_2u_2,
$$

$$
y_1 = C_1x + D_{11}u_1 + D_{12}u_2,
$$

$$
y_2 = C_2x + D_{21}u_1 + D_{22}u_2
$$

Denoting by $(A_m, B_m, C_m)$ a minimal state-space realization of the ideal model $H_m$ given in (3), by $(A_s, B_s, C_s)$ a minimal realization of the shaped system $G_s = W_2GW_1$ and by $(A_w, B_w, C_w)$ a realization of the dynamic weighting $W_3$ it follows that

$$
A = \begin{bmatrix}
A_s & 0 & 0 \\
0 & A_m & 0 \\
B_wT_sC_s & -B_wC_m & A_w
\end{bmatrix},
$$

$$
B_1 = \begin{bmatrix}
0 & -H \\
B_m & 0 \\
0 & B_wT_s
\end{bmatrix},
$$

$$
B_2 = \begin{bmatrix}
B_s \\
0 \\
0
\end{bmatrix},
$$

$$
C_1 = \begin{bmatrix}
C_s & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
$$

$$
C_2 = \begin{bmatrix}
0 & 0 & 0 \\
C_s & 0 & 0
\end{bmatrix},
$$

$$
D_{11} = \begin{bmatrix}
0 & I \\
0 & 0 \\
0 & D_wT_s
\end{bmatrix},
$$

$$
D_{12} = \begin{bmatrix}
0 \\
W_s \\
0
\end{bmatrix},
$$

$$
D_{21} = \begin{bmatrix}
I & 0 \\
0 & I
\end{bmatrix},
$$

where $T_r = \begin{bmatrix}
0 & 0 & 1
\end{bmatrix}$ selects the regulated output $\theta$ from the measurement vector $y$, namely $T_r y = \theta$ and where $H = - ZC_s$, $Z$ standing for the stabilizing solution of the Riccati equation ([15])

$$
A_sZ + ZA_s^T - ZC_s^TC_sZ + B_sB_s^T = 0.
$$

The positive weighting $W_u$ has been introduced in the matrix $D_{12}$. It weights the control variable $\delta_e$ in order to reduce its amplitude for the saturation avoidance. Solving the above $H_{\infty}$ norm
minimization problem one obtains the 2 DOF controller $K = [K_1 \quad K_2]$, representing for the PAH autopilot with the structure shown in Fig. 4.

![Diagram](image)

**FIG.4.** The 2 DOF autopilot

**4. CASE STUDY FOR THE HIRRUS UAV**

Before illustrating the design methodology presented above one notices from Fig. 5 that in the absence of the autopilot, the time response at step commands for the pitch angle is large due to the very small damping ratio of the phugoid.

![Graph](image)

**FIG.5.** Pitch angle time response to unit step command

In order to improve the flying qualities of the UAV we used the design methodology presented in the previous section. One considered the linear model (1), (2) of the Hirrus UAV and then the $H_\infty$ norm minimization problem for the system (4) have been solved using the following values for the weighting coefficients

$$W_1(s) = \frac{10^5}{100s + 1}, \quad W_2 = I_3 \quad \text{and} \quad W_e(s) = \frac{10^6(0.01s + 1)}{100s + 1}.$$

Two cases have been considered, corresponding to two different values of the weighting $W_u$ penalizing the control variable $\delta_e$, namely $W_u = 10$ and $W_u = 1000$, respectively. The time response of the error tracking for these two situations are presented in Fig. 6 and Fig. 7, respectively.
The time response control $\delta_e$ for the two cases are given in Fig. 8 and Fig. 9, respectively.

**FIG. 6.** Tracking error time response for $W_u = 10$

**FIG. 7.** Tracking error time response for $W_u = 1000$

**FIG. 8.** Control variable $\delta_e$ time response for $W_u = 10$

**FIG. 9.** Control variable $\delta_e$ time response for $W_u = 1000$
From the above plots it follows for both cases the settling time has been considerably reduced with respect to the case when the autopilot is not coupled. Moreover one also notices that when increasing the weighting $W_u$ one obtains a smaller magnitude of the control variable but the tracking error of the ideal model response increases.

5. FINAL REMARKS

A modified loop-shaping $H_\infty$ method with two degrees of freedom is presented for the design of the pitch attitude hold autopilot of an unmanned air vehicle. The numerical results and simulations indicate that the proposed approach is appropriate for acquiring robust stability, tracking of ideal model output and reduced sensitivity performances. These performances may be accomplished using a moderate complexity controller.

ACKNOWLEDGMENT

This paper has been supported by MEN-UEFISCDI, Program Partnerships, Projects PN-II-PT-PCCA- 2013-4-1349 and PN-II-PT-PCCA- 2011-3.1-1560.

REFERENCES
