# CALCULATING TECHNICAL SCATTERING FULL ERROR FOR EXTERNAL SUSPENDED BOMBS AT FREE RELEASE 

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#### Abstract

The error of aviation bombs mission use is sum of aiming error and technical scattering error. Technical scattering full error is sum of partial errors. Technical scattering full error of aviation bomb free release is determined.


Keywords: technical scattering full error, bomb

## 1. INTRODUCTION

The expression of the error of aviation bombs mission use $\xi$ can be written in the following way:

$$
\begin{equation*}
\xi=\xi_{\mathrm{a}}+\xi_{\mathrm{t}}, \tag{1}
\end{equation*}
$$

where $\xi_{\mathrm{a}}$ is the error in solving the aiming problem;

- $\xi_{\mathrm{t}}$ error caused by bombs technical scattering.

Diversions at the initial conditions of bombs movement are the main reason for technical scattering of the bomb explosion points [2, 3, 4]. Another important reason for technical scattering is diversion of ballistic bomb qualities from nominal ones [4].

Technical scattering caused by the setting is along the course of flight (in the longitudinal plane).

Technical scattering full error of the i bomb $\Delta \xi_{t}$ is presented as a sum of errors with partial reasons:

$$
\begin{equation*}
\Delta \xi_{\mathrm{ti}}=\sum_{\mathrm{j}=1}^{\mathrm{n}} \Delta \xi_{\mathrm{tij}} \tag{2}
\end{equation*}
$$

where j is a symbol of partial reason;
n - the number of reasons.
Each partial error $\Delta \xi_{\mathrm{tij}}$ is divided into a group error, equal for all bombs and expressed in shifting the whole bombs series and individual - displacing the length of the series and redistributing the bombs in the series.

Considerable partial errors are:

- $\Delta \xi_{t t}$, error of delay and "disorder" of the moment of dropping, caused by the errors of control system discharge and the errors in the time of moving at the initial section;
- $\Delta \xi_{t \theta}$, error of oscillation of bomb path, caused by diversion of bomb axis from the tangent in the process of discharge and causing angular velocity of the bomb;
- $\Delta \xi_{\text {th }}$, error, caused by different levels between points of dropping and compartment (inner suspension);
- $\Delta \xi_{\text {tv }}$, error caused by the aircraft rate in value and direction in the process of successive bomb dropping and the change in bomb velocity at the initial part of moving.

The full technical scattering error caused by the discharge process is a sum of the partial errors.

$$
\begin{align*}
\Delta \xi_{\mathrm{ti}} & =\sum_{\mathrm{j}=1}^{\mathrm{n}} \Delta \xi_{\mathrm{tij}}=\mathrm{W}_{\xi} \Delta \mathrm{t}_{\mathrm{i}}+\frac{\partial \mathrm{A}}{\partial \mathrm{C}_{\mathrm{x} \bar{\sigma}}} \Delta \mathrm{C}_{\mathrm{x} \overline{\mathrm{E}}}  \tag{3}\\
& +\frac{\partial \mathrm{A}}{\partial \mathrm{~V}} \Delta \mathrm{~V}_{\mathrm{i}}+\frac{\partial \mathrm{A}}{\partial \lambda} \Delta \lambda_{\mathrm{i}},
\end{align*}
$$

where

$$
\begin{aligned}
& \Delta V_{i}=\Delta V_{i}^{(1)}+\Delta V_{i}^{(2)} \\
& \Delta \lambda_{i}=\Delta \lambda_{i}^{(1)}+\Delta \lambda_{i}^{(2)} .
\end{aligned}
$$

## 2. MATHEMATICAL MODEL

The odd movement of each bomb at the initial part can be considered as a diversion of real values of initial parameters from predicted values.

Differences in corresponding values $x_{b}-x_{b}^{r}, y_{b}-y_{b}^{r}, \ldots \ldots \ldots$. and so on for the moment of time $t_{d}$ are diversion of real movement parameters from predicted at the surrounding aircraft area. They are caused by bomb disturbances. Outside disturbance zone and when there is a full bomb oscillation damping, real movement will be distinguished by predicted one only by difference in movement parameters at the surrounding aircraft area. That is why movement parameters at the surrounding aircraft area are considered as initial movement conditions.

Using the formula (3) we have technical scattering full error assessment for fighter bomber at horizontal flight with following data:

- airplane mass - m=15480 kg;
- airplane weight $-\mathrm{G}=\mathrm{mg}=1.5186^{*} 10^{5}$ N;
- wing area $-\mathrm{S}=34.5 \mathrm{~m}^{2}$;
- airplane rate - V=250 m/s;
- flight altitude - $\mathrm{H}=1000 \mathrm{~m}$.

The study is for practical aviation bomb P-50-75.

When there is bomb free release equations for mass center movement are [1]
$\overline{\mathrm{x}}=\left[\frac{1}{2}\left(\mathrm{n}_{\mathrm{bx}}-\mathrm{n}_{\mathrm{x}}\right) \mathrm{g}\right] \mathrm{gt}^{2}+\frac{1}{3} \omega_{\mathrm{z}}\left(\mathrm{n}_{\mathrm{by}}-\mathrm{n}_{\mathrm{y}}\right) \mathrm{gt}^{3} ;$ $\overline{\mathrm{y}}=\frac{1}{2}\left(\mathrm{n}_{\mathrm{by}}-\mathrm{n}_{\mathrm{y}}\right) \mathrm{gt}^{2}-\frac{1}{3}\left[\omega_{\mathrm{z}}\left(\mathrm{n}_{\mathrm{bx}}-\mathrm{n}_{\mathrm{x}}\right)\right] \mathrm{gtt}^{3}$.

Having in mind that [1] $\mathrm{x}_{\mathrm{b}}=\mathrm{x}_{\mathrm{a}}+\overline{\mathrm{x}}+\mathrm{x}_{0}$, $y_{b}=y_{a}+\bar{y}+y_{0}$, where $x_{0}=y_{0}=0, x_{a}=V t, y_{a}=0$ and using formula (4) we receive real parameters for bomb movement.

$$
\begin{aligned}
& \mathrm{x}_{\mathrm{b}}=\mathrm{Vt}-4.57 \mathrm{t}^{2}-0.0007 \mathrm{t}^{3} ; \\
& \mathrm{y}_{\mathrm{b}}=0.23 \mathrm{t}^{2}-0.014 \mathrm{t}^{3} ; \\
& \dot{\mathrm{x}}_{\mathrm{b}}=\mathrm{t}-9.14 \mathrm{t}-0.0021 \mathrm{t}^{2} \\
& \dot{\mathrm{y}}_{\mathrm{b}}=0.46 \mathrm{t}-0.042 \mathrm{t}^{2} .
\end{aligned}
$$

Bomb angle of pitch $\theta_{\mathrm{b}}$ and angular velocity $\dot{\theta}_{\mathrm{b}}$ are assessed by formulas [1]:

$$
\begin{equation*}
\theta_{\mathrm{b}}=\omega_{\mathrm{z}} \mathrm{t}+\mathrm{Q} \frac{\mathrm{t}^{2}}{2} ; \dot{\theta}_{\mathrm{b}}=\omega_{\mathrm{z}}+\mathrm{Qt} \tag{6}
\end{equation*}
$$

Bomb coordinates and their derivatives in the border of disturbance zone $t=t_{d}$ are:

- real coordinates

$$
\begin{aligned}
& \mathrm{x}_{\mathrm{b}}=\mathrm{Vt}_{\mathrm{d}}-4.57 \mathrm{t}_{\mathrm{d}}^{2}+0.0111 \mathrm{t}_{\mathrm{d}}^{3} \\
& \mathrm{y}_{\mathrm{b}}=-3.537 \mathrm{t}_{\mathrm{d}}^{2}-0.0143 \mathrm{t}_{\mathrm{d}}^{3} \\
& \dot{\mathrm{x}}_{\mathrm{b}}=\mathrm{V}-9.149 \mathrm{t}_{\mathrm{d}}+0.0332 \mathrm{t}_{\mathrm{d}}^{2} \\
& \dot{\mathrm{y}}_{\mathrm{b}}=-7.074 \mathrm{t}_{\mathrm{d}}-0.043 \mathrm{t}_{\mathrm{d}}^{2} \\
& \dot{\theta}_{\mathrm{b}}=\omega_{\mathrm{z}}+\mathrm{Qt}_{\mathrm{d}} ; \theta_{\mathrm{b}}=\omega_{\mathrm{z}} \mathrm{t}_{\mathrm{d}}+\mathrm{Q} \frac{\mathrm{t}_{\mathrm{d}}^{2}}{2}
\end{aligned}
$$

- predicted coordinates

$$
\begin{aligned}
& \mathrm{x}_{\mathrm{b}}^{\mathrm{r}}=\mathrm{Vt} \mathrm{t}_{\mathrm{d}} ; \mathrm{y}_{\mathrm{b}}^{\mathrm{r}}=-4.905 \mathrm{t}_{\mathrm{d}}^{2} ; \\
& \dot{\mathrm{x}}_{\mathrm{b}}^{\mathrm{r}}=\mathrm{V} ; \dot{\mathrm{y}}_{\mathrm{b}}^{\mathrm{r}}=-9.81 \mathrm{t}_{\mathrm{d}} ; \\
& \theta_{\mathrm{b}}^{\mathrm{r}}=-0.0392 \mathrm{t}_{\mathrm{d}} ; \\
& \dot{\theta}_{\mathrm{b}}^{\mathrm{r}}=-0.0392
\end{aligned}
$$

In the border of disturbance zone for coordinate $y=y_{b d}$ we define $t_{d}$ :

- real movement

$$
\mathrm{t}_{\mathrm{d}}=\sqrt{\frac{\mathrm{y}_{\mathrm{bd}}}{-3.537}},
$$

- predicted movement

$$
\mathrm{t}_{\mathrm{d}}^{\mathrm{r}}=\sqrt{\frac{\mathrm{y}_{\mathrm{bd}}}{-4.905}} .
$$

When $y_{b d}=-1.5 \mathrm{~m}, \mathrm{~V}=250 \mathrm{~m} / \mathrm{s}, \omega_{\mathrm{z}}=-$ $0.0047 \mathrm{~s}^{-2}, \mathrm{Q}=0.16$ [1] we have:

- real movement
$t_{d}=0.7976$;
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ROMANIA

$$
\begin{aligned}
\mathrm{x}_{\mathrm{b}} & =196.49 \mathrm{~m} ; \\
\mathrm{y}_{\mathrm{b}} & =-2.25 \mathrm{~m} ; \\
\dot{\mathrm{x}}_{\mathrm{b}} & =242.709 \mathrm{~m} / \mathrm{s} ; \\
\dot{\mathrm{y}}_{\mathrm{b}} & =-5.669 \mathrm{~m} / \mathrm{s} \\
\theta_{\mathrm{b}} & =0.0471 \\
\dot{\theta}_{\mathrm{b}} & =0.123 \mathrm{~s}^{-1}
\end{aligned}
$$

- predicted movement

$$
\mathrm{t}_{\mathrm{d}}^{\mathrm{r}}=0.6773
$$

$$
\mathrm{x}_{\mathrm{b}}^{\mathrm{r}}=169.321, \mathrm{y}_{\mathrm{b}}^{\mathrm{r}}=-2.25
$$

$$
\dot{\mathrm{x}}_{\mathrm{b}}^{\mathrm{r}}=250 \mathrm{~m} / \mathrm{s}, \quad \dot{\mathrm{y}}_{\mathrm{b}}^{\mathrm{r}}=-6.644 \mathrm{~m} / \mathrm{s} ;
$$

$$
\theta_{\mathrm{b}}^{\mathrm{r}}=-0.0266
$$

$$
\dot{\theta}_{\mathrm{b}}^{\mathrm{r}}=-0.0392 \mathrm{~s}^{-1} .
$$

The difference between real and predicted bomb movement in the disturbance zone is assessed by formulas:

$$
\begin{aligned}
& \Delta \mathrm{t}_{\mathrm{d}}=\mathrm{t}_{\mathrm{d}}-\mathrm{t}_{\mathrm{d}}^{\mathrm{r}} ; \\
& \Delta \mathrm{x}_{\mathrm{b}}=\mathrm{x}_{\mathrm{b}}-\mathrm{x}_{\mathrm{b}}^{\mathrm{r}}, \Delta \mathrm{y}_{\mathrm{b}}=\mathrm{y}_{\mathrm{b}}-\mathrm{y}_{\mathrm{b}}^{\mathrm{r}} ; \\
& \Delta \dot{\mathrm{x}}_{\mathrm{b}}=\dot{\mathrm{x}}_{\mathrm{b}}-\dot{\mathrm{x}}_{\mathrm{b}}^{\mathrm{r}}, \Delta \dot{\mathrm{y}}_{\mathrm{b}}=\dot{\mathrm{y}}_{\mathrm{b}}-\dot{\mathrm{y}}_{\mathrm{b}}^{\mathrm{r}} ; \\
& \Delta \theta_{\mathrm{b}}^{\mathrm{r}}=\theta_{\mathrm{b}}-\theta_{\mathrm{b}}^{\mathrm{r}} \\
& \Delta \dot{\theta}_{\mathrm{b}}=\dot{\theta}_{\mathrm{b}}-\dot{\theta}_{\mathrm{b}}^{\mathrm{r}}
\end{aligned}
$$

From where:

$$
\begin{aligned}
\Delta \mathrm{t}_{\mathrm{d}} & =0.12 \mathrm{~s} \\
\Delta \mathrm{x}_{\mathrm{b}} & =27.169 \mathrm{~m} \\
\Delta \dot{\mathrm{x}}_{\mathrm{b}} & =-7.29 \mathrm{~m} / \mathrm{s} \\
\Delta \dot{\mathrm{y}}_{\mathrm{b}} & =0.975 \mathrm{~m} / \mathrm{s} \\
\Delta \theta_{\mathrm{b}}^{\mathrm{r}} & =0.0737 \\
\Delta \dot{\theta}_{\mathrm{b}} & =0.162 \mathrm{~s}^{-1}
\end{aligned}
$$

Technical scattering full error is assessed by formula (3) and airplane ground velocity $\mathrm{W}_{\xi}$ is assessed by formula:

$$
\begin{equation*}
\mathrm{W}_{\xi}=\mathrm{V}+\mathrm{U}, \tag{7}
\end{equation*}
$$

where $U$ is the wind velocity. We assume that $\mathrm{U}=10 \mathrm{~m} / \mathrm{s}$. Then airplane ground velocity is $W_{\xi}=260 \mathrm{~m} / \mathrm{s}$.

Partial error $\Delta \xi_{\mathrm{tW}}$ :

$$
\Delta \xi_{\mathrm{tW}}=\mathrm{W}_{\xi} \Delta \mathrm{t}_{\mathrm{d}}=31.2 \mathrm{~m}
$$

Resistance coefficient increasing $\Delta \mathrm{C}_{\mathrm{xb}}$ is received by formula:

$$
\Delta \mathrm{C}_{\mathrm{xb}}=3 \% \mathrm{C}_{\mathrm{xb}}=0.6146 * \frac{3}{100}=0.0184
$$

After solving the ballistic problem we receive the partial error, caused by $\Delta \mathrm{C}_{\mathrm{xb}}$ :

$$
\begin{aligned}
\Delta \xi_{\mathrm{tCxb}} & =\frac{\partial \mathrm{A}}{\partial \mathrm{C}_{\mathrm{xb}}} \Delta \mathrm{C}_{\mathrm{xb}}= \\
& =813.587 * 0.0184=14.97 \mathrm{~m}
\end{aligned}
$$

When the flight is horizontal, errors $\Delta \mathrm{V}^{(1)}=0, \Delta \lambda^{(1)}=0$, as aviation target system automatically reads the rate of flight, bomb weight is slightly less than airplane weight and the angle $\lambda=0$.

Errors $\Delta \mathrm{V}^{(2)}$ and $\Delta \lambda^{(2)}$, caused by bomb movement in the disturbance zone are received by formulas:

$$
\begin{aligned}
& \Delta \mathrm{V}^{(2)}=\Delta \dot{\mathrm{x}}_{\mathrm{b}}= \\
& =\mathrm{V}_{\mathrm{bx}}^{\mathrm{r}}\left(\mathrm{t}_{\mathrm{d}}\right)-\mathrm{V}_{\mathrm{bx}}\left(\mathrm{t}_{\mathrm{d}}\right)=-7.29 \mathrm{~m} / \mathrm{s} ; \\
& \quad \Delta \lambda^{(2)}=\frac{\Delta \dot{\mathrm{y}}_{\mathrm{b}}-\Delta \dot{\mathrm{x}}_{\mathrm{b}} \operatorname{tg} \lambda}{\dot{x}_{\mathrm{b}}} \cos ^{2} \lambda= \\
& \quad=0.22^{0} .
\end{aligned}
$$

Partial error is calculated by formula (3):

$$
\begin{align*}
& \Delta \xi_{\mathrm{tv}}=\frac{\partial \mathrm{A}}{\partial \mathrm{~V}} \Delta \mathrm{~V}^{(2)}+\frac{\partial \mathrm{A}}{\partial \lambda} \Delta \lambda^{(2)}=  \tag{10}\\
& =-19.9-14=-33.9 \mathrm{~m} .
\end{align*}
$$

Technical scattering full error is calculated by formula (3):

$$
\begin{aligned}
\Delta \xi_{\mathrm{tj}}= & \sum_{\mathrm{j}=1}^{4} \Delta \xi_{\mathrm{tj}}=\mathrm{W}_{\xi} \Delta \mathrm{t}_{\mathrm{d}}+\frac{\partial \mathrm{A}}{\partial \mathrm{C}_{\mathrm{xb}}} \Delta \mathrm{C}_{\mathrm{xb}}+ \\
& +\frac{\partial \mathrm{A}}{\partial \mathrm{~V}} \Delta \mathrm{~V}^{(2)}+\frac{\partial \mathrm{A}}{\partial \lambda} \Delta \lambda^{(2)}= \\
= & 31.2-14.97-19.9-14=-17.67 \mathrm{~m} .
\end{aligned}
$$

## 3. CONCLUSIONS

From so done calculations for partial errors for the used example with accepted conditions for bomb release we see that partial error $W_{\xi} \Delta t_{d}$ has positive sign. The others $\frac{\partial \mathrm{A}}{\partial \mathrm{C}_{\mathrm{xb}}} \Delta \mathrm{C}_{\mathrm{xb}}, \frac{\partial \mathrm{A}}{\partial \mathrm{V}} \Delta \mathrm{V}^{(2)}, \frac{\partial \mathrm{A}}{\partial \lambda} \Delta \lambda^{(2)} \quad$ have negative sign, i.e. there is compensation for partial errors and technical scattering full error is reduced.

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