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AEROENGINE COMBUSTION INSTABILITY – AN ANALYTICAL EVALUATION

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Abstract: *Combustion processes are sensitive to fluctuations of pressure, density and temperature of the environment. Even slow changes of those quantities affect the energy released according to rules that can be deduced from the behavior for steady combustion. Combustion instabilities normally occur in frequency ranges such that genuine dynamical behavior is significant. That is, the transient changes of energy release do not follow precisely in phase with imposed changes of a flow variable such as pressure. A fluctuation of burning produces local changes in the properties of the flow. Those fluctuations propagate in the medium and join with the global unsteady field in the chamber. The dynamical response of the medium converts the local fluctuations to global behavior. In this paper are presented some results and remarks about combustion instabilities.*

Keywords: *propulsion, combustion, aircraft engine*

1. INTRODUCTION

Chemical propulsion systems depend fundamentally on the conversion of energy stored in molecular bonds to mechanical energy of a vehicle in motion. The first stage of the process, combustion of oxidizer and fuel takes place in a vessel open only to admit reactants and to exhaust the hot products. Higher performance is achieved by increasing the rate of energy release per unit volume. A useful strategy, particularly for applications to flight, is reduction of the average temperature at which combustion takes place. Generation of NO by the thermal or Zeldovich' mechanism is then reduced. Lower combustion temperature may be achieved by operating under lean conditions, when the flame stabilization processes tend to be unstable. Fluctuations of the flame cause fluctuations of energy release, which in turn may produce fluctuations of pressure, exciting acoustical motions in the chamber. The simplest assumption is that combustion processes behave as a first order dynamical system characterized by a single time delay or relaxation time. There are three main reasons that the classical view of acoustics is a good first approximation to wave propagation in combustion chamber:

- the Mach number of the average flow is commonly small, so convective and refractive effects are small;

- the exhaust nozzle is choked, incident waves are efficiently reflected, so for small Mach number the exit plane appears to be nearly a rigid surface;

- in the limit of small amplitude disturbances, it is a fundamental result for compressible flow that any unsteady motion can be decomposed into three independent modes of propagation, of which one is acoustic. The other two modes of motion are vertical disturbances, the dominant component of turbulence, and entropy waves.

The most obvious evidence that combustion instabilities are related to classical acoustic resonances is the common observation that frequencies measured in tests agree fairly well with those computed with classical formulas.

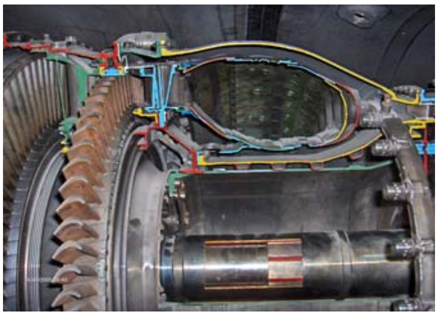

Fig. 1. Combustion chamber

Generally, the frequency *f* of a wave equals its speed of propagation, *a*, divided by the wavelength λ

$$f = \frac{a}{\lambda} \qquad (1)$$

The wavelengths of the organ-pipe modes are proportional to the length *L* of the pipe, those of modes of motion in transverse planes of a circular chamber (fig. 1) are proportional to the diameter *D*, and so forth.

There are two basic implications of the conclusion that these formulas seem to predict observed frequencies fairly well: evidently the geometry is a dominant influence on the special structure of the instabilities and we can define some sort of average speed of sound in the chamber (fig. 2), based on an approximation to the temperature distribution.

2. CHARACTERISTICS OF COMBUSTION INSTABILITIES

It is a general result of the theory of linear systems that if a system is unstable, a small disturbance of an initial state will grow exponentially in time: amplitude of disturbance $\approx e^{\alpha_g \cdot t}$ where $\alpha_g > 0$ is called growth constant. If a disturbance is linearly stable, then its amplitude decays exponentially in time, being proportional to $e^{-\alpha_d \cdot t}$ and $\alpha_d > 0$ is the decay constant. Having maximum amplitude $\hat{p}_0$ in one cycle of a linear oscillation the pressure is

$$p'(t) = \hat{p}_0 \cdot e^{\alpha_g (t - t_0)} \qquad (2)$$

where $\hat{p}_0$ is the amplitude at time $t - t_0$. If p_1' and p_2' are the peak amplitudes at time t_1 and t_2, we have

$$\frac{\hat{p}_2}{\hat{p}_2} = \frac{p'(t = t_2)}{p'(t = t_1)} = \frac{e^{\alpha_g (t_2 - t_0)}}{e^{\alpha_g (t_1 - t_0)}} = e^{\alpha_g (t_2 - t_1)} \qquad (3)$$

$$\log \frac{\hat{p}_2}{\hat{p}_1} = \alpha_g (t_2 - t_1) \qquad (4)$$

In practice, $t_2 - t_1$ is usually taken equal to the period τ, the time between successive positive (or negative) peaks.

There is really only one problem to solve: find the growth and decay constants, and the frequencies of the modes. Typically, both the frequency and the mode shape for small-amplitude motions in a combustion chamber are so little different from their values computed classically as to be indistinguishable by measurement in operating combustors.

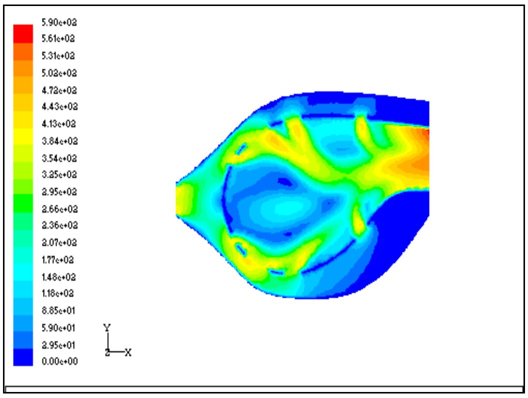

Fig. 2. Velocity field

The linear stability problem is really concerned with calculations of the growth and decay constants for the modes corresponding to the classical acoustic resonances. An arbitrary small amplitude motion can, in principle, be synthesized with the results, but that calculation is rarely required for practical applications.

Results for the net growth or decay constant have been the central issue in both theoretical and practical work. In combustors, processes causing growth of disturbances and those causing decay act simultaneously. Hence an unstable disturbance is characterized by a net growth constant that can be written $\alpha = \alpha_g - \alpha_d$. Because the problem is linear, the growth constants can quite generally be expressed as a sum of the contributions due to processes accounted for in the formulation, as for example:

$$\alpha = \alpha_g - \alpha_d = (\alpha)_{combustion} + (\alpha)_{nozzle} + (\alpha)_{mean\ flow} + (\alpha)_{structure} + \ldots \quad (5)$$

The stability boundary – the locus of parameters making the boundary between unstable $(\alpha > 0)$ and stable $(\alpha < 0)$ oscillations – is defined by $\alpha = 0$. That statement is a formal statement of the physical condition that the energy gained per cycle should equal the energy lost per cycle: $\alpha_g = \alpha_d$

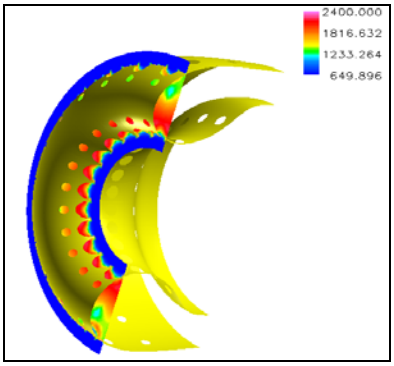

Fig. 3. Temperature field

By the definition of α, both the pressure and velocity oscillations have the time dependence

$$\begin{cases} p' \approx e^{\alpha \cdot t} \cos(\omega \cdot t) \\ u' \approx e^{\alpha \cdot t} \sin(\omega \cdot t) \end{cases} \quad (6)$$

multiplied by their spatial distributions. The acoustic energy density is the sum of the local kinetic energy, proportional to $(u')^2$, and potential energy. If we assume that the period of oscillation, $\tau = 2\pi/\omega$ is much smaller than the decay rate, $1/\alpha$, and the values of these functions averaged over a cycle of the oscillations are proportional to $e^{2\alpha \cdot t}$, the acoustic energy density is itself proportional to $e^{2\alpha \cdot t}$. Integrating over the total volume of the chamber we find that the total averaged energy is $E = E_0 e^{2\alpha \cdot t}$, where E_0 is a constant depending on the average flow properties, the temperature flow field (fig. 3) and the geometry. We find the result

$$2\alpha = \frac{1}{E_0} \frac{dE}{dt} \quad (7)$$

Another elementary property is that $1/\alpha$ is the time required for the amplitude of oscillation to decay to $1/e$ of some chosen initial value. Also, the fractional change of the peak value in one cycle of oscillation $(t_2 - t_1 = \tau = 2\pi/\omega)$ is

$$|p'_2| - |p'_1| = \delta |p'|_m \approx e^{\alpha t_1} - e^{\alpha t_2} = e^{\alpha t_2}\left(e^{\alpha(t_1 - t_2)} - 1\right) \quad (8)$$

where $|\ |_m$ denotes the magnitude of the peak amplitude. Assuming that the fractional change in one period τ is small, so

$$e^{\alpha(t_1 - t_2)} \approx 1 + \alpha(t_1 - t_2) = 1 + \alpha \cdot \tau \quad (9)$$

The amplitude itself is approximately proportional to $e^{\alpha t_2}$ or $e^{\alpha t_1}$ and we can write the fractional change as

$$\frac{\delta |p'|_m}{|p'|_m} \approx \alpha \cdot \tau = \frac{\alpha}{f} \quad (10)$$

where f is the frequency in cycles per second, $f = \frac{1}{\tau}$. The dimensionless ratio f/α is a convenient measure of the growth or decay of an oscillation. According to the interpretation noted above, f/α is the number of cycles required for the maximum amplitudes of oscillation to decay to $1/e$ or grow to ‘e’ times an initial value.

3. NONLINEAR BEHAVIOR

We may anticipate that nonlinear behavior may be regarded in first approximations as an extension of the view of linear behavior because the frequency varies little, remaining close to a value computed classically for a natural resonance of the camber, and the growth of the peak amplitude during the initial transient period is quite well approximated by the rule for a linear stability. Thus the behavior is distinguishable from that of a classical linear oscillator with damping, and having a single degree of freedom. The governing equation for the free motion of a simple mass (m), spring (k) and dashpot (r) is

$$m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=0 \qquad (11)$$

It is surely tempting to model a linear combustion instability by identifying the pressure fluctuation, p', with the displacement x of the mass. Then upon dividing this equation by m and replacing x by p', we have

$$\frac{d^2p'}{dt^2}+2\alpha\frac{dp'}{dt}+\omega_0^2p'=0 \qquad (12)$$

where $2\alpha=r/m$ and the undampened natural frequency is $\omega_0=\sqrt{k/m}$.

According to the theory of classical acoustics for a sound wave, we may identify both kinetic energy per unit mass, proportional to the square of the acoustic velocity u' and potential energy per unit mass, proportional to the square of the acoustic pressure p'. The acoustic energy per unit volume is

$$\frac{1}{2}\left(\bar{\rho}u'^2+\frac{p'^2}{\bar{\rho}\bar{a}^2}\right) \qquad (13)$$

where $\bar{\rho}$ and $\bar{a}$ are the average density and the speed of sound. This expression corresponds to the formula for the energy of a simple oscillator

$$\frac{1}{2}\left(m\dot{x}^2+kx^2\right) \qquad (14)$$

Both the velocity and the pressure fluctuations have spatial distributions such that the boundary condition of no velocity normal to a rigid wall is satisfied. Hence the local pressure p' in the equation (12) must depend on position as well as time. However, the frequency ω_0 depends on the geometry of the entire chamber and according to equation (7) we should be able to interpret 2α as the fractional rate of change of the averaged energy in the entire volume.

Locally in the medium the spring constant is supplied by the compressibility of the gas and the mass participating in the motion is proportional to the density of the undisturbed medium. When the procedure of spatial averaging is applied, both the compressibility and the density are weighted by the appropriate spatial structure of the acoustical motion. As a result, the damping constant and the natural frequency are expressed in terms of global quantities characterizing the fluctuating motion throughout the chamber. So, we can represent

$$p'_n=\bar{p}\,y_n(t)\,\phi_n(\vec{r}) \qquad (15)$$

where $\bar{p}$ is the mean pressure and $\phi_n(\vec{r})$ is the spatial structure of the classical acoustic mode identified by the index $(\ )_n$. Hence the typical equation of motion is

$$\frac{d^2y_n}{dt^2}+2\alpha_n\frac{dy_n}{dt}+\omega_n^2=0 \qquad (16)$$

The constants α_n and ω_n contain the influences of all linear processes distinguishing the oscillation in a combustion chamber from the corresponding unperturbed classical motion governed by the equation

$$\frac{d^2y_n}{dt^2}+\omega_{n0}^2y_n=0 \qquad (17)$$

if dissipation of energy is ignored. Because damping in a mechanical system causes a frequency shift, the actual frequency is not equal to the unperturbed value, ω_{n0}.

For combustion chamber it is convenient to regard the linear perturbing process as a force $F_n(y_n,\dot{y}_n)$, so equation (17) is written

$$\frac{d^2 y_n}{dt^2} + \omega_{n0}^2 y_n = F_n^L(y_n, \dot{y}_n) \quad (18)$$

The force F_n^L consists of two terms, one representing the damping of the mode and one the frequency shift:

$$F_n^L = -\Delta\omega_n^2 y_n + 2\alpha_n \dot{y}_n \quad (19)$$

According to classical acoustic theory, a closed chamber of gas at rest has an infinite number of normal or resonant modes. The spatial structures (mode shapes) and resonant frequencies are found as solutions to an eigenvalue problem. A general motion in the chamber, having any spatial structure, can then be represented as a linear superposition of the normal modes. The process of spatial averaging, leading to equation (17), amounts to representing any motion as an infinite collection of simple oscillators, one associated with each of the normal modes. That interpretation holds as well for equation (18) except that now each mode may suffer attenuation $(\alpha_n < 0)$ or excitation $(\alpha_n > 0)$.

Determining the linear stability of a system comes down to computing the value of the constant α. Assume that only one mode is active and the driving force is entirely due to fluctuations of the rate of heat $\dot{Q}'$ provided to the flow, in simplest form the equation for the amplitude is

$$\frac{d^2 y}{dt^2} + \omega_1^2 y = (\gamma - 1)\int \frac{\partial \dot{Q}'}{\partial t} \phi dV \quad (20)$$

where $\phi(r)$ is the spatial distribution of the pressure for the mode defined so the fluctuation is $p' = \bar{p} y \phi(r)$. Suppose that the heat release rate is sensitive only to pressure and write its fluctuation as

$$\dot{Q}' = \frac{\dot{Q}'}{p'} p' = Rp' = \bar{p} R y \phi \quad (21)$$

where R is the response function, having dimensions of inverse time

$$[R] = \frac{[Energy/Volume]\frac{1}{t}}{[Energy/Volume]} = t^{-1}$$

the substitution of (21) in (20) leads to a formula for α:

$$\frac{d^2 y}{dt^2} + \omega_1^2 y = \left[(\gamma - 1)\bar{p}\int R\phi^2 dV\right]\frac{dy}{dt} = 2\alpha \frac{dy}{dt} \quad (22)$$

and α is proportional to the response function

$$\alpha = \frac{\gamma - 1}{2}\bar{p}\int R\phi^2 dV \quad (23)$$

The equation governing the amplitude is

$$\frac{d^2 y}{dt^2} - 2\alpha \frac{dy}{dt} + \omega_1^2 = 0 \quad (24)$$

with the solution

$$y(t) = Ae^{\alpha t}\cos(\omega_2 t + \varphi) \quad (25)$$

where A and φ are constants and $\omega_2^2 = \omega_1^2 - \alpha^2$.

If α is positive, the oscillation is driven by the response of the heat release to the pressure fluctuations.

The essential idea in all applications of the time lag is that a finite interval – the lag – exists between the time when an element of propellant enters the chamber and the time when it burns and releases its chemical energy. Suppose that at time t the pressure in the chamber suddenly decreases, causing an increase in the flow of propellant through the injector, the increased mass burns at some later time $t + \tau$, where τ is the time lag.

4. CONCLUSIONS

Whatever the system, most combustion instabilities involve excitation of the acoustic modes, for which there are an infinite number for any type of chamber. The values of the frequencies are functions primarily of the geometry and of the speed of sound. These modes are unstable and depend on the balance of energy supplied by the exciting mechanisms and extracted by the dissipating processes.

The presence of the combustion processes and a mean flow field are not accounted for explicitly, but it is necessary to include a good approximation to the boundary condition applied at the exhaust nozzle, particularly if the average Mach number is not small.

The most important measure of the perturbations is a Mach number, $\overline{M}_r$ characterizing the mean flow; for many significant processes, α / f equals $\overline{M}_r$ times a constant of order unity, so the measured value of α / f is an initial indication of the validity of the view that a combustion instability can be regarded as a motion existing because of relatively weak perturbations.

REFERENCES

1. Ananthkrishnan, N., Deo, S., Culick, F.E.C., *Reduced-Order Modeling and Dynamics of Nonlinear Acoustic Waves in a Combustion Chamber,* Combustion Science and Technology, Vol. 177, No. 2 (2005).
2. Krebs, W., Flohr, P., Prade, B., Hoffman, S., *Thermoacoustic Stability Chart for High Intense Gas Turbine Combustion Systems*, Combustion Science and Technology, Vol. 174, No. 2 (2003).
3. Rotaru, C., *Aspects regarding combustion chamber dynamics for turbojet engines,* International Conference on Military Technologies, Brno, Czech Republic, (2009).
4. Rotaru, C., Arghiropol, A., Barbu, C., *Some aspects regarding possible improvements in the performances of the aircraft engines*, Proceedings of the 6th IASME/WSEAS International Conference on Fluid Dynamics and Aerodynamics, Greece, (2008).
5. Rotaru, C., Soloi, A., Safta, D., *Analysis of compressor-combustion chamber dynamics for aircraft* engines, Proceedings of the International Conference on Mathematical Problems in Engineerings, Aerospace and Sciences, Genoa, Italy, (2008).
6. Rotaru, C., Safta, D., Barbu, C., *Aspects regarding internal flow in combustion chamber of turbojet engines*, Proceedings of the 5th IASME/WSEAS International Conference on Fluid Dynamics and Aerodynamics, Greece, (2007).
7. Shauer, F., Stutrud, J., Bradley, R., *Detonation Initiation Studies and Performance Results from Pulsed Detonation Engine Applications*, AIAA Paper (2001).
8. Shauer, F., Bradley, R., Hoke, J., *Interaction of a Pulsed Detonation Engine with a Turbine*, AIAA Paper (2003).