ON GENERAL CONFORMAL METRICAL N-LINEAR CONNECTIONS ON DUAL BUNDLE OF K-TANGENT BUNDLE

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Abstract: In the present paper we give the transformations for the coefficients of an N-linear connection on dual bundle of k-tangent bundle, \( T^*kM \), by a transformation of a nonlinear connection on \( T^*kM \). Starting from the notion of conformal metrical d-structure we define the notion of general conformal metrical N-linear connection on dual bundle of k-tangent bundle. We determine the set of all general conformal metrical N-linear connections, in the case when the nonlinear connection is fixed and we find important particular cases. Finally we find the transformations group of these connections.

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1. INTRODUCTION

The notion of Hamilton space was introduced by Acad. R. Miron in [5], [6]. The Hamilton spaces appear as dual via Legendre transformation, of the Lagrange spaces. The differential geometry of the dual bundle of \( k \) – osculator bundle was introduced and studied by Acad. R. Miron [10]. In the present section we keep the general setting from Acad. R. Miron [10], and subsequently we recall only some needed notions. For more details see [10].

Let \( M \) be a real n-dimensional \( C^\infty \) manifold and let \( (T^*kM, \pi^*k, M) \), \((k \geq 2, k \in N)\) be the dual bundle of k-osculator bundle (or k-cotangent bundle), where the total space is \( T^*kM = T^{*k-1}M \times T^*M \). (1)

Let \( (x', y^{(1)}_i, ..., y^{(k-1)}_i, p_i) \), \( i = 1, 2, ..., n \), be the local coordinates of a point \( u = (x, y^{(1)}_i, ..., y^{(k-1)}_i, p) \in T^*kM \) in a local chart on \( T^*kM \). The change of coordinates on the manifold \( T^*kM \) is...
Let \( N \) be another nonlinear connection on \( T^kM \), with the local coefficients
\[
\left( N^i_j \right) \left( x, y^{(1)}, \ldots, y^{(k-1)}, p \right), \ldots, N^j_i \left( x, y^{(1)}, \ldots, y^{(k-1)}, p \right), N_y \left( x, y^{(1)}, \ldots, y^{(k-1)}, p \right) \big|_{i,j = 1,2,\ldots,n}.
\]
Then there exists the uniquely determined tensor fields \( A^i_j, A^i_j \in T^k_1 \left( T^kM \right) \), \((\alpha = 1, \ldots, k-1)\) and \( A_y \in T^k_0 \left( T^kM \right) \), such that
\[
N^i_j = N^i_j - A^i_j, \quad \left( \alpha = 1, \ldots, k-1 \right),
\]
\[
N_y = N_y - A_y, \quad \left( i,j = 1,2,\ldots,n \right).
\]

Conversely, if \( N^i_j \) and \( A^i_j \), \((\alpha = 1, \ldots, k-1)\), respectively \( N_y \) and \( A_y \) are given, then \( N^i_j \), \((\alpha = 1, \ldots, k-1)\), respectively \( N_y \), given by (8) are the coefficients of a nonlinear connection.

**Theorem 1** Let \( N \) and \( \overline{N} \) be two nonlinear connections on \( T^kM, (k \geq 2, k \in N) \), with local coefficients
\[
\left( N^i_j \right) \left( x, y^{(1)}, \ldots, y^{(k-1)}, p \right), \ldots, N^j_i \left( x, y^{(1)}, \ldots, y^{(k-1)}, p \right), N_y \left( x, y^{(1)}, \ldots, y^{(k-1)}, p \right) \big|_{i,j = 1,2,\ldots,n}.
\]

If \( D \Gamma \left( N \right) = \left( H^i_j, H^i_j, C^i_j \right) \) and
\[
D \overline{\Gamma} \left( \overline{N} \right) = \left( \overline{H}^i_j, \overline{C}^i_j \right), \quad \left( \alpha = 1, \ldots, k-1 \right)
\]
are the local coefficients of two \( N-\), respectively \( \overline{N} -\)linear connections, \( D \), respectively \( \overline{D} \) on the differentiable manifold \( T^kM, (k \geq 2, k \in N) \), then the transformation
\[ N \rightarrow \overline{N}, \text{ given by (8) of nonlinear connections} \]

implies for the coefficients of the \( \overline{N} \)-linear connection \( D\Gamma(\overline{N}) = \left\{ \overline{H}^i_{\ jh}, \overline{C}^i_{\ jh}, \overline{C}^j_{\ ih} \right\} \), the following relations

\[
\begin{align*}
H^i_{\ sj} &= H^i_{\ sj} + A^m_{\ (k-2)} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \ldots + \right] \\
&\quad + N^l_{\ (k-3) m} C^i_{\ sl} + \ldots + N^l_{\ (1) m} N^l_{\ (1) m} C^i_{\ sl} + \ldots + \\
&\quad + N \ldots N C_{\ (1) (1) (k-1)} \\
&\quad + A^m_{\ (2)} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] \\
&\quad + \ldots + A^m_{\ (k-2)} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] \\
&\quad + A^m_{\ (k-1)} \left[ C^i_{\ sm} + A_m_{\ jh} C^i_{\ sl} + \right] \\
&\quad + \ldots + A^m_{\ (k-2)} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] \\
&\quad + \ldots + A^m_{\ (k-1)} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right]
\end{align*}
\]

\[ + \ldots + A^m_{\ (k-3) m} C^i_{\ sl} + \ldots + N \ldots N C_{\ (1) (1) (k-1)} \]

\[ + \ldots + A^m_{\ (k-3) m} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] + \]

\[ + \ldots + A^m_{\ (k-2) m} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] + \]

\[ + \ldots + A^m_{\ (k-1) m} \left[ C^i_{\ sm} + A_m_{\ jh} C^i_{\ sl} + \right] + \]

\[ + \ldots + A^m_{\ (k-3) m} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] + \]

\[ + \ldots + A^m_{\ (k-2) m} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] + \]

\[ + \ldots + A^m_{\ (k-1) m} \left[ C^i_{\ sm} + A_m_{\ jh} C^i_{\ sl} + \right] + \]

\[ + \ldots + A^m_{\ (k-3) m} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] + \]

\[ + \ldots + A^m_{\ (k-2) m} \left[ C^i_{\ sm} + N^l_{\ (k-1)} m C^i_{\ sl} + \right] + \]

\[ + \ldots + A^m_{\ (k-1) m} \left[ C^i_{\ sm} + A_m_{\ jh} C^i_{\ sl} + \right] + \]

where “\( \nabla \)” denotes the \( h \)-covariant derivative with respect to \( D\Gamma(\overline{N}) \)

**Theorem 2** Let \( N \) and \( \overline{N} \) be two nonlinear connections on \( T^k M, (k \geq 2, k \in N) \), with local coefficients

\[
\left( N^j_{\ i} (x, y^{(1)}, \ldots, y^{(k-1)}, p) \right) \ldots \left( N^j_{\ i} (x, y^{(1)}, \ldots, y^{(k-1)}, p) \right) = \ldots
\]

\[
\left( \overline{N}^j_{\ i} (x, y^{(1)}, \ldots, y^{(k-1)}, p) \right) \ldots \left( \overline{N}^j_{\ i} (x, y^{(1)}, \ldots, y^{(k-1)}, p) \right)
\]

\[ i, j = 1, 2, \ldots, n, \text{ respectively.} \]

If

\[ D\Gamma(N) = \left\{ H^i_{\ jh}, C^i_{\ jh}, C^j_{\ ih} \right\} \]

and

\[ D\Gamma(\overline{N}) = \left\{ \overline{H}^i_{\ jh}, \overline{C}^i_{\ jh}, \overline{C}^j_{\ ih} \right\}, \quad (\alpha = 1, \ldots, k-1) \]

are the local coefficients of two \( N \)-, respectively \( \overline{N} \)-linear connections, \( D \), respectively \( \overline{D} \) on the differentiable manifold \( T^k M, (k \geq 2, k \in N) \), then there exists only one system of tensor fields
such that
\[
\overline{N}^j = N_j - A^j, \quad (\alpha = 1, \ldots, k-1),
\]
\[
N_{ij} = N_{ij} - A_{ij},
\]
\[
\overline{H}^j_{ij} = H^j_{ij} + A^m_{(2)} \left[ C^i_{(2) m} + N^i_{(1) m} C^i_{sl} + \ldots + N^i_{(2-3) m} C^i_{st} + \ldots + N^i_{(k-3) m} C^i_{st} + \ldots \right] +
\]
\[
A^m_{(2)} \left[ C^i_{(2) m} + N^i_{(1) m} C^i_{st} + \ldots + N^i_{(2-3) m} C^i_{st} + \ldots + N^i_{(k-3) m} C^i_{st} + \ldots \right] +
\]
\[
\overline{N}^i_{(2)} = N^i_{(2)} - A^i_{(2)},
\]
\[
\overline{N}^i_{(1)} = N^i_{(1)} - A^i_{(1)},
\]
\[
\overline{N}^i_{(k-3)} = N^i_{(k-3)} - A^i_{(k-3)},
\]
\[
\overline{N}^i_{(k-1)} = N^i_{(k-1)} - A^i_{(k-1)},
\]
\[
\overline{C}^i_{(2) sj} = C^i_{(2) sj} + A^m_{(2)} \left[ C^i_{(2) sm} + N^i_{(1) m} C^i_{sr} + \ldots + N^i_{(2-3) m} C^i_{sr} + \ldots + N^i_{(k-3) m} C^i_{sr} + \ldots \right] +
\]
\[
A^m_{(2)} \left[ C^i_{(2) sm} + N^i_{(1) m} C^i_{sr} + \ldots + N^i_{(2-3) m} C^i_{sr} + \ldots + N^i_{(k-3) m} C^i_{sr} + \ldots \right] +
\]
\[
\overline{C}^i_{(2-3) sj} = C^i_{(2-3) sj} + A^m_{(2-3)} \left[ C^i_{(2-3) sm} + N^i_{(1) m} C^i_{sr} + \ldots + N^i_{(2-3) m} C^i_{sr} + \ldots + N^i_{(k-3) m} C^i_{sr} + \ldots \right] +
\]
\[
A^m_{(2-3)} \left[ C^i_{(2-3) sm} + N^i_{(1) m} C^i_{sr} + \ldots + N^i_{(2-3) m} C^i_{sr} + \ldots + N^i_{(k-3) m} C^i_{sr} + \ldots \right] +
\]
\[
\overline{C}^i_{(k-1)} = C^i_{(k-1)} - D^i_{(1) sj},
\]
\[
\overline{C}^i_{(k-3)} = C^i_{(k-3)} - D^i_{(2-3) sj},
\]
\[
\overline{C}^i_{(k-1)} = C^i_{(k-1)} - D^i_{(k-1) sj},
\]

where $\overline{\cdot}$ denotes the $h$-covariant derivative with respect to $D \Gamma(N)$.

In the particular case when we have the same nonlinear connection $N$, that is $A^i_{(\alpha) j} = 0$, $(\alpha = 1, \ldots, k-1), (k \geq 2, k \in N)$ and $A_{ij} = 0$, we obtain the set of transformations of $N$-linear connections corresponding to the same nonlinear connection $N$ given by
\[
\overline{H}^j_{ij} = H^j_{ij} - B^j_{ij},
\]
\[
\overline{N}^i_{(\alpha) j} = N^i_{(\alpha) j} - D^i_{(\alpha) sj},
\]
\[
(\alpha = 1, \ldots, k-1), (k \geq 2, k \in N)
\]

3. General Conformal Metrical $N$-Linear Connections in the Hamilton Space of Order $k, k \geq 2, k \in N$

Let $H^{(k)n} = (M, H)$ be a Hamiltonian space of order $k, k \geq 2, k \in N$, and let $N$ be the canonical nonlinear connection of the space $H^{(k)n}$ (\cite{10}, p.192).

We consider the adapted basis
\[
\left\{ \frac{\delta}{\delta x^i}, \frac{\delta}{\delta \tilde{x}^{(1)i}}, \ldots, \frac{\delta}{\delta \tilde{x}^{(k-1)i}}, \frac{\delta}{\delta \tilde{p}_j} \right\}
\]
and its dual basis
\[
\left\{ \delta x^i, \delta \tilde{x}^{(1)i}, \ldots, \delta \tilde{x}^{(k-1)i}, \delta \tilde{p}_j \right\}
\]
determined by $N$ and by the distribution $W_k$. Let
\[
g^{ij}(x, y(1), \ldots, y^{(k-1)}, p) = \frac{1}{2} \frac{\partial^2 H}{\partial \tilde{p}_i \partial \tilde{p}_j}
\]
be the fundamental tensor of the space $H^{(k)n}$ [10].

The d-tensor field $g^{ij}$ being nonsingular on $T^* M = T^* M - \{0\}$ (where 0 is the null section of the projection $\pi^k$) there exists a d-tensor field $g_{ij}$ covariant of order 2, symmetric,
uniquely determined, at every point \( u \in T^k M \), by \( g^j_j = \delta^j_i \) \(^{(13)}\).

**Definition 1** ([10]) An \( N \)-linear connection \( D \) is called compatible to the fundamental tensor \( g^{ij} \) of the Hamiltonian structure on \( T^k M \), or it is metrical if \( g^{ij} \) is covariant constant (or absolute parallel) with respect to \( D \), i.e.,

\[
g^{ij} |_h = 0, \quad g^{ij} |^h = 0, \quad (\alpha = 1, \ldots, k - 1) \tag{14}\]

The operators of Obata's type are given by \( \Omega_{hkl}^\alpha = \frac{1}{2}(\delta^\alpha_h \delta^i_k - g_{hk} g^{ij}) \), \( \Omega^{\alpha h} = \frac{1}{2}(\delta^\alpha_h \delta^i_k + g_{hk} g^{ij}) \) \(^{(15)}\).

**Proposition 1** The operators of Obata's type are covariant constant with respect to any metrical \( N \)-linear connection, \( D \),

\[
\Omega_{hkl}^\alpha = 0, \quad \Omega^{\alpha h} = 0, \quad (\alpha = 1, \ldots, k - 1).
\]

Let \( S_2(T^k M) \) be the set of all symmetric \( d \)-tensor fields, of the type \((0,3)\) on \( T^k M \), \( k \geq 2, k \in N \). As is easily shown, the relations for \( a_{ij}, b_{ij} \in S_2(T^k M) \) defined by:

\[
(a_{ij} \approx b_{ij}) \quad \Leftrightarrow \quad ((\exists) \lambda(x, y^{(1)}, \ldots, y^{(k-1)}, p) \in F(T^k M), \quad a_{ij}(x, y^{(1)}, \ldots, y^{(k-1)}, p) = e^{2\lambda(x, y^{(1)}, \ldots, y^{(k-1)}, p)} b_{ij}(x, y^{(1)}, \ldots, y^{(k-1)}, p)) \tag{16}
\]

is an equivalence relation on \( S_2(T^k M) \).

**Definition 2** The equivalent class \( \hat{g} \) of \( S_2(T^k M) \) to which the fundamental \( d \)-tensor field \( g_{ij} \) belongs, is called a general conformal metrical \( d \)-structure on \( T^k M \).

Thus

\[
\hat{g} = \{g^\prime | g^\prime_{ij}(x, y^{(1)}, \ldots, y^{(k-1)}, p) = e^{2\lambda(x, y^{(1)}, \ldots, y^{(k-1)}, p)} g_{ij}(x, y^{(1)}, \ldots, y^{(k-1)}, p), (\alpha = 1, \ldots, k - 1) \}
\]

is called general conformal metrical \( N \)-linear connection, \( D \), with local coefficients

\[
D\Gamma(N) = \left(H^i_{\alpha jh}, C^i_{\alpha jh}, C^h_{ij} \right), (\alpha = 1, \ldots, k - 1),
\]

for which there exists the 1-form \( \omega \),
\[
\alpha = \omega dx^i + \omega_j \delta^{(1)}_{ij} + \cdots + \omega_j \delta^{(k-1)}_{ij} + \omega^i \delta^i_j,
\]
such that
\[
\begin{align*}
g_{ijh}^{(a)} &= 2 \omega_h g_{ij}, \quad g_{ijh}^{(a)} = \hat{\omega}_h g_{ij}, \\
g_{ijh}^{(h)} &= 2 \omega_h g_{ij}, \quad g_{ijh}^{(h)} = \hat{\omega}_h g_{ij},
\end{align*}
\]
where \( | \) \( h \), \( | \) \( h \), denote the \( h- \), \( v_{\alpha} \)- and \( w_k \)- covariant derivatives with respect to \( D \), \((\alpha = 1,\ldots,k-1)\) is called conformal metrical \( N \)-linear connection, with respect to the conformal metrical \( d \)-structure \( \hat{g} \), corresponding to the 1-form \( \omega \) and it is denoted by: \( D\Gamma(N,\omega) \).

**Proposition 2** If \( D\Gamma(N,\omega) \)
\[
= \left( H^{i}_{jkh}, C^{i}_{(\alpha) jh}, C^{jh}_{(\alpha)} \right) \quad (\alpha = 1,\ldots,k-1) \text{ are the local coefficients of a conformal metrical } N \text{-linear connection in } T^k M, \text{ with respect to the conformal metrical structure } \hat{g} , \text{ corresponding to the 1-form } \omega , \text{ then}
\]
\[
g_{ijh}^{(a)} = -2 \omega_h g_{ij}^{(a)}, \quad g_{ijh}^{(h)} = -2 \omega_h g_{ij}^{(h)}, \quad g_{ijh}^{(a)} = -2 \omega_h g_{ij}^{(a)}, \quad \omega_{\alpha} = \omega + d\lambda .
\]

For any representative \( g' \in \hat{g} \) we have

**Theorem 3** For \( g' = e^{2\lambda} g_{ij} \), a conformal metrical \( N \)-linear connection with respect to the conformal metrical structure \( \hat{g} \), corresponding to the 1-form \( \omega \) in \( T^k M, D\Gamma(N,\omega) \), satisfies
\[
g_{ijh}^{(a)} = 2 \omega_h g'_{ij}, \quad g_{ijh}^{(h)} = 2 \omega_h g'_{ij}, \quad g_{ijh}^{(a)} = 2 \omega_h g'_{ij}, \quad \omega' = \omega + d\lambda .
\]

Since in Theorem 3 \( \omega' = 0 \) is equivalent to \( \omega = d(\lambda) \), we have

**Theorem 4** A conformal metrical \( N \)-linear connection with respect to \( \hat{g} \), corresponding to the 1-form \( \omega \) in \( T^k M, D\Gamma(N,\omega) \), is metrical with respect to \( g' \in \hat{g} \), i.e. \( g_{ijh}^{(a)} = g_{ijh}^{(h)} = 0 \) if and only if \( \omega \) is exact.

We shall determine the set of all general conformal metrical \( N \)-linear connections, with respect to \( \hat{g} \), corresponding to the same nonlinear connection \( N \).

Let
\[
0 \quad D\Gamma(N) = \left( H^{i}_{jkh}, C^{i}_{(\alpha) jh}, C^{jh}_{(\alpha)} \right), \quad (\alpha = 1,\ldots,k-1)
\]
be the local coefficients of a fixed \( N \)-linear connection \( D \), where
\[
(N_i^j(x,y^{(1)},...,y^{(k-1)}),p), N_{ij}^k(x,y^{(1)},...,y^{(k-1)}),p),
\]
\((\alpha = 1,\ldots,k-1),(i,j = 1,2,\ldots,n)\)
are the local coefficients of the nonlinear connection \( N \). Then any \( N \)-linear connection, \( D \), with the local coefficients
\[
D\Gamma(N) = \left( H^{i}_{jkh}, C^{i}_{(\alpha) jh}, C^{jh}_{(\alpha)} \right), (\alpha = 1,\ldots,k-1),
\]
can be expressed in the form (11).

Using the relations (20), (11), (6) and the Theorem 1 given by R. Miron in ([4]) for the case of Finsler connections we obtain

**Theorem 5** Let \( D \) be a given \( N \)-linear connection, with local coefficients \( D\Gamma(N) \)
\[
= \left( H^{i}_{jkh}, C^{i}_{(\alpha) jh}, C^{jh}_{(\alpha)} \right) \quad (\alpha = 1,\ldots,k-1) .
\]
The set of all general conformal metrical \( N \)-linear connections, with respect to \( \hat{g} \), corresponding to the same nonlinear connection \( N \) with local coefficients
\[
D\Gamma(N) = \left( H^{i}_{jkh}, C^{i}_{(\alpha) jh}, C^{jh}_{(\alpha)} \right), (\alpha = 1,\ldots,k-1)
\]
is given by
\[
\begin{align*}
H^{i}_{jkh} &= H^{i}_{jkh} + \frac{1}{2} g^{mj}_{ij} (g_{mjh}^{(h)} - K_{mjh}) + \Omega_{\beta}^{\gamma} X_j^{\gamma}, \\
C^{i}_{(\alpha) jh} &= C^{i}_{(\alpha) jh} + \frac{1}{2} g^{mj}_{ij} (g_{mjh}^{(h)} - Q_{mjh}^{(h)} + \Omega_{\beta}^{\gamma} Y_j^{\gamma}) \quad (\alpha = 1,\ldots,k) \\
C^{jh}_{(\alpha) i} &= C^{jh}_{(\alpha) i} + \frac{1}{2} g^{mj}_{ij} (g_{mjh}^{(h)} - Q_{mjh}^{(h)} + \Omega_{\beta}^{\gamma} Z_j^{\gamma}), \quad (\alpha = 1,\ldots,k-1)
\end{align*}
\]
(21)
where \( w^h_k \) denote the covariant derivatives with respect to \( D \), \( X_{jh}^{\alpha}, Y_{jk}^{\alpha}, Z_{i}^{jk} \) are arbitrary d-tensor fields and \( K_{ijh}, Q_{ijh}, Q_{ijh}^{h} \) are arbitrary d-tensor fields of the types \((0,3), (0,3), (2,1)\) respectively, with the properties \((19), (\alpha = 1,\ldots,k-1)\).

**Particular cases**

1. If we take

\[
K_{ijh} = 2\omega_{h} g_{ij}, Q_{ijh} = 2\omega_{h} g_{ij}, (\alpha = 1,\ldots,k-1),
\]

\[
Q_{ijh}^{h} = 2\omega^{k} g_{ij}
\]

in Theorem 5, we obtain

**Theorem 6** Let \( D \) be a given \( N \)-linear connection, with local coefficients \( D\Gamma(N) = \left(\begin{array}{c} H_{ijh}^{\alpha} \\ C_{ijh}^{\alpha} \\ C_{ijh}^{\alpha} \end{array}\right) \) \((\alpha = 1,\ldots,k-1)\). The set of all conformal metrical \( N \)-linear connections with respect to \( \hat{g} \), corresponding to the 1-form \( \omega \), with local coefficients \( D\Gamma(N,\omega) = \left(\begin{array}{c} H_{ijh}^{\alpha} \\ C_{ijh}^{\alpha} \\ C_{ijh}^{\alpha} \end{array}\right) \) \((\alpha = 1,\ldots,k-1)\) is given by

\[
H_{ijh}^{\alpha} = H_{ijh}^{\alpha} + \frac{1}{2} g^{mn}_{(\alpha)}(g_{mh}^{\alpha} - 2\omega_{h} g_{mj}^{\alpha}) + \Omega_{ijh}^{\alpha} X_{rh}^{s},
\]

\[
C_{ijh}^{\alpha} = C_{ijh}^{\alpha} + \frac{1}{2} g^{mn}_{(\alpha)}(g_{mj}^{\alpha} - 2\omega_{h} g_{mj}^{\alpha}) + \Omega_{ijh}^{\alpha} Y_{rh}^{s},
\]

\[
C_{ijh}^{\alpha} = C_{ijh}^{\alpha} + \frac{1}{2} g^{mn}_{(\alpha)}(g_{mh}^{\alpha} - 2\omega_{h} g_{mh}^{\alpha}) + \Omega_{ijh}^{\alpha} Z_{rh}^{s},
\]

\((\alpha = 1,\ldots,k-1)\).

2. If \( X_{jh}^{\alpha} = Y_{jk}^{\alpha} = Z_{i}^{jk} = 0 \), in Theorem 5 we have an example of general conformal metrical with respect to \( \hat{g} \):

**Theorem 7** Let \( D \) be a given \( N \)-linear connection, with local coefficients \( D\Gamma(N) = \left(\begin{array}{c} H_{ijh}^{\alpha} \\ C_{ijh}^{\alpha} \\ C_{ijh}^{\alpha} \end{array}\right) \) \((\alpha = 1,\ldots,k-1)\). Then the following \( N \)-linear conection \( K \), with local coefficients

\[
K\Gamma(N) = \left(\begin{array}{c} H_{ijh}^{\alpha} \\ C_{ijh}^{\alpha} \\ C_{ijh}^{\alpha} \end{array}\right) \) \((\alpha = 1,\ldots,k-1)\), given by (23) is general conformal metrical with respect to \( \hat{g} \).
\[ H^i_{\ jh} = H^i_{\ jh} + \frac{1}{2} g^{im} (g_{mj}^0 |_h^0 - K_{mhj}^0), \]

\[ C^i_{\ (a) \ jh} = C^i_{\ (a) \ jh} + \frac{1}{2} g^{im} (g_{mj}^0 |_h^0 - Q_{\ (a) \ mjh}^0), \]

\[ C^i_{\ jh} = C^i_{\ jh} + \frac{1}{2} g^{mi} (g_{mj}^0 |_h^0 - \dot{Q}_{\ mi}^k), \]

\[ \alpha = 1, \ldots, k - 1, \]

where \( g_{\ jh} \) and \( v_\alpha \) denote the \( h \)- and \( v_\alpha \)-covariant derivatives with respect to \( D \), and \( K_{\ jh}, Q_{\ (a) \ jh}, \dot{Q}_{\ jh} \) are arbitrary d-tensor fields of the types \((0,3)\), \((0,3)\) and \((2,1)\) respectively, with the properties \((19)\), \((\alpha = 1, \ldots, k - 1)\).

3. If we take a general conformal metrical N-linear connection with respect to \( \hat{g} \) as \( D \), in Theorem 5 we have

**Theorem 8** Let \( D \) be on \( T^k M \) a fixed general conformal metrical N-linear connection with respect to \( \hat{g} \), with the local coefficients

\[
D \Gamma(N) = \left( H^i_{\ jh}, C^i_{\ (a) \ jh}, C^i_{\ jh} \right) \quad (\alpha = 1, \ldots, k - 1).
\]

The set of all general conformal metrical N-linear connections, with respect to \( \hat{g} \), with local coefficients

\[
D \Gamma(N) = \left( H^i_{\ jh}, C^i_{\ (a) \ jh}, C^i_{\ jh} \right), \quad (\alpha = 1, \ldots, k - 1)
\]

given by

\[
H^i_{\ jh} = H^i_{\ jh} + \Omega^r_{\ jh} X^s_r,
\]

\[
C^i_{\ (a) \ jh} = C^i_{\ (a) \ jh} + \Omega^r_{\ (a) \ jh} Y^s_r, \quad (\alpha = 1, \ldots, k - 1),
\]

\[
C^i_{\ jh} = C^i_{\ jh} + \Omega^r_{\ jh} Z^s_r,
\]

where \( X^i_{\ jh}, Y^i_{\ (a) \ jh}, Z^i_{\ jh} \) are arbitrary d-tensor fields, \((\alpha = 1, \ldots, k - 1)\).

4. If \( K_{\ jh} = Q_{\ (a) \ jh} = \dot{Q}_{\ jh} \) \( h = 0 \), \((\alpha = 1, \ldots, k - 1)\) in Theorem 5 we obtain the set of all metrical N-linear connection in the case when the nonlinear connection is fixed, result given in ([10]).

**Theorem 9** The mappings determined by

\[
(23), D \Gamma(N) \rightarrow D \Gamma(N) \text{together with the composition of these mappings is an abelian group.}
\]

**REFERENCES**


