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A GAME THEORY MODEL OF STOCK EXCHANGE MARKET MANIPULATION

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Abstract: *The role of the stock exchange market is to finance the national economy, therefore anti-crisis measures must be taken by the state to support and encourage the investments on the financial markets. An important measure which can be taken by the National Securities Commission is to charge higher fines on market abuse which should lead to fewer cases of abuses by discouraging manipulators, leading to a higher level of trust from behalf of the local and foreign investors, due to higher protection imposed by the authorities. The hereto paper develops a static game of complete information, between the National Securities Commission (the stock exchange market regulator) and an investor which tries to manipulate the stock exchange market. The model presents an investor which chooses between two trategies: to manipulate the market or not to manipulate the market, and the National Securities Commission which chooses between two strategies: to investigate the market abuse or not to investigate it.*

Keywords: *stock market manipulation, market regulation, mixed strategies*

1. INTRODUCTION

The role of the stock exchange market is to finance the national economy, therefore anti-crisis measures must be taken by the state to support and encourage the investments on the financial markets. An important measure which can be taken by the representative of the state on the stock exchange market, namely the market regulator or "watchdog" called the National Securities Commission, is to charge higher fines on market abuse which should lead to fewer cases of abuses, leading to a higher level of trust from behalf of the local and foreign investors, due to higher protection imposed by the authorities, thus attracting more and more investors and funds on the local stock market.

One of the anti-crisis measures taken by the state during the global crisis in 2009, in order to support the investments on the financial markets, was to completely cutt off the tax on capital gain for the 2009 fiscal year. On the other hand, the Bucharest Stock Market decided to reduce trading commissions, especially commissions on buying transactions, and, based on the positive trend of global stock exchange markets' indices, the local stock market showed a strong recovery, at least in the first four months of 2009.

At the end of 2010, the National Securities Commission's (CNVM) representatives stated that the institution is preparing legislative changes in order to raise the value of fines charged for market abuse, by calculating them as a percentage of the turnover.

On the other hand, the fiscal legislation regarding the tax on capital gains for 2010 fiscal year was verry confusing, as in the third quarter the Government has modified taxation starting with July 1st, leading to a steap dropp in the number of deals on the Bucharest Stock Exchange market, to around 45,000 in September, from 91,000 in June and 130,000

in May. Fiscal legislation issues have a great impact upon investors' decisions, therefore the state must choose verry carefully among different strategies, because the final purpose of the state is the social welfare which can be also obtained by encouraging investments.

Lately, there has been a constant development of the theoretical literature on market manipulation, starting with Hart and Kreps (1986) [1], Vila (1987, 1989) [2], Allen and Gale (1992) [3], Benabou and Laroque (1992) [4], and Jarrow (1992, 1994) [5, 6] who were among the first researcher to study market manipulation. Subsequent contributions include Bagnoli and Lipman (1996) [7], Chakraborty and Yilmaz (2004) [8]. Vitale (2000) [9] considers manipulation in foreign exchange markets, while Van Bommel (2003) [10] shows the impact of rumors in price manipulation.

Allen and Gale (1992) [3] propose a classification scheme for models of manipulation.

The hereto paper develops a static game model of complete information, between the National Securities Commission (the stock exchange market regulator) and an investor which is tempted to manipulate the stock exchange market. The players of the game are: an investor who chooses between two strategies: to manipulate the market or not to manipulate it, and the National Securities Commission which also chooses between two strategies: to investigate the market abuse on the market or not to investigate it.

Section 2 describes the stock market manipulative strategies which are taken into consideration in this paper, while Section 3 introduces the static game model in complete information. In order to describe the game model, we define a normal-form representation of the game, and try to find a strictly dominated strategy and a mixed strategy, which will be interpreted in terms of a player's uncertainty about what the other player will do.

2. STOCK MARKET MANIPULATIVE STRATEGIES

Manipulation can occur through actions taken by insiders that influence the stock price (accounting and earnings manipulation), or by the release of false information or rumors in press or on the Internet, which influence stock prices. Also large block trades can influence prices, therefore by purchasing a large amount of stock, a trader can drive the price up, and profit on the back of the price increase [11]. Note that in this paper, we reffer only to the regulated stock exchange market, which is supervised by the National Securities Commission.

Potentially informed parties are corporate insiders, brokers, large shareholders and market makers, which are likely to be manipulators, while illiquid stocks are more likely to be manipulated and manipulation increases stock volatility [12]. An investor, especially an informed trader, has to balance the short term profit from the trade with the long term effect his trade has on the beliefs of the market and on future profits, thus a strategy is manipulative if it involves the informed trader undertaking a trade in any period which gives him strictly negative short term profit in order to manipulate the beliefs of the market regarding his private information, enabling him to recoup the short term losses and more in the future [8]. In our model, the investor's payoff in case he does not manipulate the market is considered to be $\pi \in \Re$, while the additional payoff generated by manipulating the market is $r \in \Re, r > 0$, called informational rent.

The manipulation described above implies the following elements, for a successful market manipulation [13]:

a) Access to a large number of potential investors, ideally at low cost.

b) Anonymity, or market manipulators might otherwise be revealed.

c) Scalability, meaning the ability of the manipulator to duplicate rumors on a large scale.

d) Time to accomplish a manipulation quickly because the danger of exposure increases with the period of time a scheme takes.

e) Impact, meaning that the people who hear the rumor should be motivated to act by trading on the false information.

3. STATIC GAME MODEL OF COMPLETE INFORMATION

3.1 Description of the model. The description of the game is best ilustrated by the normal-form representation of the game, in which each player simultaneously chooses a strategy, and the combination of strategies chosen by the players determines a payoff for each player. The problem analysed by this paper refers to an investor (INV) (any investor on the stock exchange market, often an insider) who has the possibility and the means to manipulate the market. The only one who can stop the investor from manipulating the market is the National Securities Commission (NSC), which can investigate the cases of market abuse and the investors' trading strategies.

We assume that the game is of complete information, because the players of the game know the strategies and the gains obtained by the other player, as the NSC can easily find out the profit of any investor on the stock exchange market, with the help of the institutions which operate the trades and by interogating the intermediaries on the stock exchange markets, regarding their clients, as all the trades operated on the regulated stock exchange market are introduced by the intermediaries autorized and supervised by the National Securities Commission. On the other hand, the investors on the market know the level of fines which the National Securities Commission is allowed to charge, because it is provisioned by the capital market laws and regulations. We also assume the correctiveness of the NSC, therefore in case the NSC investigates and the investor manipulates the market, he will be caught and fined, while if the investor is inocent and the NSC investigates the market, he will not be fined. Also, we do not take into consideration the corruption cases, in which the employees or management of the NSC are payed by the investor not to investigate the market.

We consider a market for one asset, and we assume not only that the players are rational, but also that both players know about the other that he is rational, and that both players know that the other player knows that he is rational.

In our model, the investor's payoff in case he does not manipulate the market is considered to be $\pi \in \Re$, while the additional payoff generated by manipulating the market is $r \in \Re, r > 0$, called informational rent, his gain being affected by the fine charged by the NSC in case it investigates the market. The NSC's gain comes from commissions charged from the participants on the capital market noted as $com \in \Re, com > 0$ which is tightly related to the number of transactions operated by the investors, and from fines charged on market manipulation, $\alpha \in \Re$. In case the investor manipulates the market and the NSC does not investigate, the gain obtained by the NSC will be reduced with a weight $\alpha \in (0,1)$ due to the loss of trust from behalf of other investors on the market, which leave the market not properly controled and supervised. The investigation process implies an expense $c > 0$.

Although in a normal-form game the players choose their strategies simultaneously, this does not imply that the players necessarily act simultaneously, but each player chooses his acton without knowledge of the others'choices.

The game can be represented in the following bi-matrix, Fig. 1:

Probabilities	**NSC**	**p**	**1-p**
INV	Strategy	I	NI
q	M	$\pi + r - a$ $com + a - c$	$\pi + r$ $\alpha \cdot com$
1-q	NM	π $com - c$	π com

Fig. 1 Players'payoff bi-matrix

3.2. Solving the model. In order to eliminate the temtation of the investor to manipulate the market, we must see if the strategy can be dominated by the strategy of non manipulating the market. This happens when for each feasible combination of the other players' strategies the payoff from manipulating the market is strictly less than the payoff from not manipulating the market.

The payoff of the investor according with the strictly dominated strategy will be:

$$U_{INV}(M,\cdot) = (\pi + r - a, \pi + r) \quad (1)$$

$$U_{INV}(NM,\cdot) = (\pi, \pi) \quad (2)$$

After comparing the payoffs, we can state that in order for the investor not to manipulate the market, it is necessary for the variables to meet the following conditions:

$$\begin{cases} \pi > \pi + r - a \\ \pi > \pi + r \end{cases} \Rightarrow \begin{cases} a > r \\ r < 0 \end{cases} \quad (3)$$

Due to the fact that $r > 0$ was set at the beginning to be positive, because the informational rent derived from manipulating the market is supposed to be positive, there is no dominated strategy in the model.

In order to solve the problem, the mixed strategy will be used, which implies one player's uncertainty about what the other player will do, by assigning a probability distribution to it. Therefore, a mixed strategy for the investor is the probability distribution *(q, 1-q)*, where *q* is the probability for which the NSC beliefs that the investor will manipulate the market, and *1-q* is the probability for which the NSC beliefs that the investor will not manipulate the market. A mixed strategy for the NSC is the probability distribution *(p, 1-p)*, where *p* is the probability for which the investor beliefs that the NSC will investigate the market abuse, and *1-p* is the probability for which the investor beliefs that the NSC will not investigate the market.

The expected payoffs for each strategy will be computed, taking into consideration the pobability distribution attached, as follows:

$$E(U_{INV}(M,\cdot)) = p \cdot (\pi + r - a) + (1 - p) \cdot (\pi + r) = -p \cdot a + \pi + r \quad (4)$$

$$E(U_{INV}(NM,\cdot)) = p \cdot \pi + (1 - p) \cdot \pi = \pi \quad (5)$$

In order to find out the value of the probability distribution for which the investor is indifferent between the two strategies, the expected payoffs must be equalized:

$$-p \cdot a + \pi + r = \pi \Rightarrow p = \frac{r}{a} \quad (6)$$

According with the investor's beliefs, the best response of the investor is not to manipulate the market in case $p < \frac{r}{a}$, and the best response of the investor is to manipulate the market in case $p > \frac{r}{a}$, where *p* represents the belief of the investor that the NSC will investigate.

In order to find out the value of the probability distribution for which the NSC is indifferent between the two strategies, we compute the value of q:

$$E(U_{NSC}(\cdot, I)) = q \cdot (com + a - c) + (1 - q) \cdot (com - c) = q \cdot a + com - c \quad (7)$$

$$E(U_{NSC}(\cdot, NI)) = q \cdot a \cdot com + (1 - q) \cdot com = com + q \cdot com \cdot (\alpha - 1) \quad (8)$$

In order to find out the value of the probability distribution for which the NSC is indifferent between the two strategies, the expected payoffs must be equalized as follows:

$$q \cdot a + com - c = com + q \cdot com \cdot (\alpha - 1) \Rightarrow q = \frac{c}{a + com \cdot (1 - \alpha)} \quad (9)$$

According with the NSC's beliefs, the best response of the NSC is not to investigate the market abuse in case $q < \frac{c}{a + com \cdot (1 - \alpha)}$, and the best response of the NSC is to investigate

the market in case $q > \frac{c}{a + com \cdot (1-\alpha)}$, where q represents the beliefs of the NSC that the investor will manipulate the market price.

Therefore, the mixed strategy is the following:

$$\left(\left(\frac{c}{a + com \cdot (1-\alpha)}, \frac{a + com \cdot (1-\alpha) - c}{a + com \cdot (1-\alpha)} \right), \left(\frac{r}{a}, \frac{a-r}{a} \right) \right)$$

3.3. Results. By analysing the senzitivity of the model to the variables, we can conclude that when the fine (a) set by the NSC is very high compared to the informational rent which the investor should receive by manipulating the market, $a \to \infty$ the probability $p \to 0$, therefore the NSC in tempted not to investigate the market abuse. On the other hand, in case the informational rent is close to the fine charged by the NSC for manipulating the market, $r \to a$ the probability $p \to 1$, therefore the NSC will be tempted to investigate the market abuse. We can observe that r must always be smaller than a in order to have a mixed strategy.

If we look at the variables which influence the probability for the investor to manipulate the market, we can state that the investor will be tempted to manipulate the market in case that $c \to a + com \cdot (1-\alpha), q \to 1$, but if $a + com \cdot (1-\alpha) \to \infty$ the investor will not manipulate, $q \to 0$ the market due to either high fines, or low trust from the other investors on the market which would determine the NSC to look for incomes derived from fines, instead of commissions from the market participants.

4. CONCLUSIONS & ACKNOWLEDGMENT

4.1. Conclusions. The National Securities Commission plays an important role on the stock exchange market and therefore on the local economy, because its decisions to sustain investments by protecting the investors interests and to assure a fair and healthy capital market can boost economy growth by attracting higher investments. An important measure which can be taken by the National Securities Commission is to charge higher fines on market abuse which should lead to fewer cases of market manipulation, leading to a higher level of trust from behalf of the local and foreign investors, due to higher protection imposed by the authorities.

The hereto paper develops a static game of complete information between the National Securities Commission and an investor which tries to manipulate the stock exchange market. The model presents an investor which chooses between two trategies: to manipulate the market or not to manipulate the market, and the National Securities Commission which chooses between two strategies: to investigate the market abuse or not to investigate it. The result is a set of probability distributions and restrictions which lead to indifferent reactions showed by the players, from which we can start to analyse the players' reactions to the other player's strategie. The main problem of the model is that the informational rent obtained by the investor who manipulates the market is very hard to compute by the NSC, because it represents the additional value which the investor obtains by manipulating the market, compared to the payoff that he should have gained in case he would have not manipulated the market, therefore the stock prices history would have been another, one unknown.

4.2. Future work. Further studies should be performed by adapting the model to a dynamic game in incomplete information, and computing the Bayesian equilibrium, and to compare the results with the ones in the present paper. Furthermore, study cases should be performed in order to establish how much close to the reality is the model presented in this paper.

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